Department of Mathematics, University of Wisconsin-Madison

Math 567 — Midterm Exam 2 — Spring 2025

NAME : (as it appears on Canvas)

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INSTRUCTIONS:

Time: **90 minutes**

- This exam contains 9 questions some with multiple parts, 7 pages (including the cover) for the total of 50 points. Read the problems carefully and budget your time wisely.
- **NO CALCULATORS** or other electronic devices are to be used. Turn off your phone so as to not disturb others.
- Please present your solutions in a clear manner. Cross out any writing that you do not wish to be graded.
- Justify your steps.
- If you use an additional page for a particular problem, be sure to **CLEARLY** indicate this on the problem's page so I know to look further.
- Please write your name on every page.
- You can safely assume that all unknown quantities in this exam, e.g. a, b, c, n, x, y, are always the integers.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	5	5	5	5	5	5	5	5	10	50
Score:										

1. (5 points) Find the Legendre symbol $\left(\dfrac{15}{101}\right)$.

2. (5 points) Express $-\dfrac{19}{51}$ as the finite simple continuous fraction.

3. (5 points) Show that for every prime p there is $n \in \mathbb{Z}$ such that $(n^2-2)(n^2-3)(n^2-6) \equiv 0 \pmod{p}$.

4. (5 points) Evaluate the infinite simple continuous fraction $[0, \overline{1, 2, 1}]$.

5. (5 points) Find all integer solutions for equation

$$x^2 + y^2 = 2z^2.$$

6. (5 points) Find all integer solutions for equation

$$x^2 + y^2 = 3z^2.$$

7. (5 points) Suppose that N is a nonzero integer and d is non-square positive integer. Prove that if $x^2 - dy^2 = N$ has one integer solution, then it has infinitely many.

8. (5 points) Let α be irrational number with partial convergents $\frac{p_n}{q_n}$. Use the relation

$$\alpha = \frac{r_{n+1}p_n + p_{n-1}}{r_{n+1}q_n + q_{n-1}}$$

to show that

$$\left|\alpha - \frac{p_n}{q_n}\right| < \left|\alpha - \frac{p_{n-1}}{q_{n-1}}\right|.$$

9. Let E be the elliptic curve

$$y^2 = x^3 + 2x + 4 \pmod{5}.$$

(a) (5 points) Write down all the points of $E(\mathbb{Z}/5\mathbb{Z})$.

(b) (5 points) Suppose $P = (0, 2)$. Find coordinates of $6P$.

SCRATCH PAPER - DO NOT REMOVE FROM YOUR EXAM.
SCRATCH WORK WILL NOT BE GRADED