

Department of Mathematics, University of Wisconsin-Madison
Math 475 — Midterm Exam 1 — Fall 2023

NAME : (as it appears on Canvas)

EMAIL: @wisc.edu

PROFESSOR: MIKHAIL IVANOV

INSTRUCTIONS:

Time: **90 minutes**

Please write your name on every page

Read the problems carefully and budget your time wisely.

No calculators or other electronic devices, please. **Turn off your phone.**

Please present your solutions in a clear manner. Justify your steps.

A numerical answer without explanation cannot get credit.

Cross out the writing that you do not wish to be graded on.

You must use correct notation to receive full credit.

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	20	15	10	10	10	10	10	100

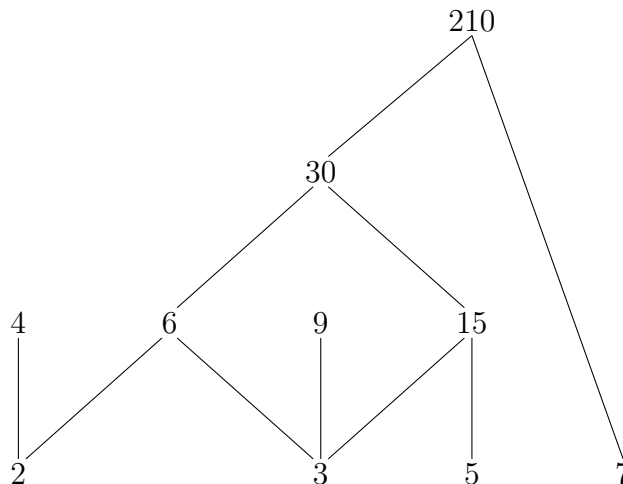
1. Consider a word

$$\mathbb{S} = \text{WISCONSIN}$$

- (a) (5 points) How many 2-permutations of \mathbb{S} are there (simplify your answer)?
- (b) (5 points) How many permutations of \mathbb{S} are there?
- (c) (5 points) 2 letters were randomly chosen from \mathbb{S} . Determine the probability that there is a vowel among them.

2. **On this page, only the answer will be graded. MARK YOUR ANSWER CLEARLY. But you do not need to justify your work.**

Consider the following partial order on the set $X = \{2, 3, 4, 5, 6, 7, 9, 15, 30, 210\}$: $x \leq_R y$ if and only if y is divisible by x (i.e. x is a divisor of y). Hasse diagram is drawn.



- (a) (4 points) How many minimal elements are there?
- (b) (4 points) Find the largest chain.
- (c) (4 points) Find the largest antichain.
- (d) (4 points) Find the smallest chain partition.
- (e) (4 points) Find the smallest antichain partition.

3. (a) (10 points) Construct permutations of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ whose inversion sequence is 3, 5, 1, 2, 1, 2, 0, 0.
- (b) (5 points) Construct the inversion sequence of the permutation 7, 3, 5, 1, 4, 8, 2, 6.

First Name: _____

Last Name: _____

4. (10 points) A chess player plans to train for his next match by playing at least one game each day for 90 days, but at most five games over any period of three consecutive days. Show that there must be some period of consecutive days during which he plays 29 games.

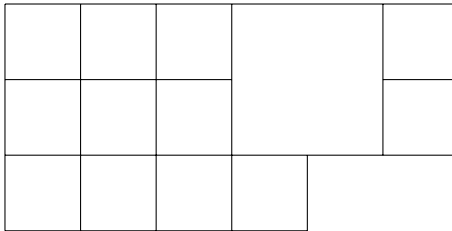
5. (10 points) Give an example of a noncyclic Gray code of order 3.

6. (10 points) List all 3-subsets of $\{1, 2, 3, 4, 5, 6\}$ in the lexicographic order.

First Name: _____

Last Name: _____

7. (10 points) How many walks are there from the lower left corner to the upper right corner taking upwards and rightwards steps only? (note the excluded portions):



8. (10 points) Determine the number of 9-combinations of the multiset

$$\{1 \cdot a, \infty \cdot b, \infty \cdot c, \infty \cdot d\}.$$