

Department of Mathematics, University of Wisconsin-Madison
Math 475 — Final Exam — Fall 2023

NAME : (as it appears on Canvas)

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INSTRUCTIONS:

Time: **120 minutes**

Please write your name on every page.

Read the problems carefully and budget your time wisely.

No calculators or other electronic devices, please. **Turn off your phone.**

Please present your solutions in a clear manner. Justify your steps.

A numerical answer without explanation cannot get credit.

Cross out the writing that you do not wish to be graded on.

You must use correct notation to receive full credit.

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	8	12	10	8	10	10	12	10	10	10	100

1. (8 points) In how many ways can 12 indistinguishable apples and 1 orange be distributed among 3 volleyball players in such a way that each player gets at least one fruit?

First Name: _____

Last Name: _____

2. Let

$$S = \text{XXXYYYZZZZ}.$$

(There are three X's, three Y's, and four Z's).

- (a) (4 points) How many permutations of the letters of S are there?
- (b) (4 points) How many permutations are there, if no two X's can be consecutive?
- (c) (4 points) How many permutations are there, if all three of the Y's have to be consecutive?

3. (10 points) Determine the number of permutations of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ in which no even numbers are in their natural positions.

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4. (8 points) Write the generating function for the number of ways to make n dollars if you can only use 1 dollar, 5 dollar, 20 dollar, and 50 dollar bills.

5. (10 points) Solve the recurrence relation

$$a_n = 8a_{n-1} - 16a_{n-2} + 9$$

for $n \geq 2$ with initial values $a_0 = 0$, $a_1 = 1$.

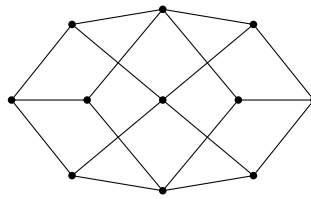
First Name: _____

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6. (10 points) How many non-isomorphic graphs (including disconnected) on 12 vertices with degree sequence $(2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2)$? Describe all of them.

7. Let G be the graph below.

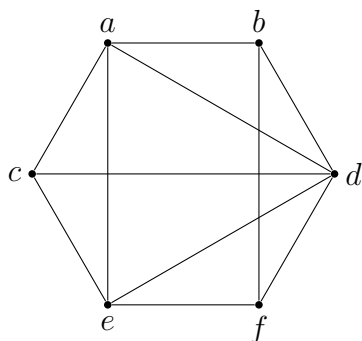
- (a) (6 points) Does G has the Hamiltonian path?
- (b) (6 points) Does G has the Hamiltonian cycle?



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8. (10 points) Find the chromatic number of the graph G below and proper coloring of $\chi(G)$ colors.



9. (10 points) Let G denote the group of symmetries for the regular 6-gon. Find the cycle index P_G .

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10. (10 points) Find the number of necklaces that have 19 beads with colors red, white, and blue.