

## Midterm 1

Name:

Student ID number:

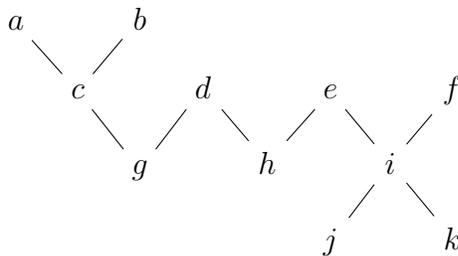
- This is exam for sections 001 and 003, instructor **Mikhail Ivanov**.
- **There are 9 problems on the exam, some of them have multiple parts.**
- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. **Turn off your phone.**
- You do not have to carry out complicated numerical computations, but you should simplify your answer if it is possible with reasonable effort. In particular, geometric series must be evaluated.
- Please present your solutions in a clear manner. Justify your steps. A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

| Problem      | Points      |
|--------------|-------------|
| 1            | /10         |
| 2            | /10         |
| 3            | /20         |
| 4            | /10         |
| 5            | /10         |
| 6            | /10         |
| 7            | /10         |
| 8            | /10         |
| 9            | /10         |
| <b>Total</b> | <b>/100</b> |

1. Consider  $9 \times 9$  chessboard and 9 rooks, of which five are red and four are blue. Suppose you place the rooks on the board in non-attacking positions at random. What is the probability that the red rooks are in rows 1, 3, 5, 7, 9?

2. There is an even number  $2n$  of people at a party and they talk together in pairs, with everyone talking to someone ( $n$  pairs). In how many ways can the  $2n$  people be talking like this?

3. Let  $(X, \leq)$  be a poset with given Hasse diagram:



- (a) How many elements are in  $X$ ?
- (b) Find all minimal elements and all maximal elements of the poset.
- (c) Determine a chain of largest size and a partition of  $X$  into the smallest number of antichains.
- (d) Determine an antichain of largest size and a partition of  $X$  into the smallest number of chains.

4. Construct the permutation of  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  whose inversion sequence is 2, 5, 5, 0, 2, 1, 1, 0.

5. Determine the reflected Gray code of order 4.

6. Suppose 70 students are studying 11 different subjects, and any subject is studied by at most 15 students. Show that there are at least three subjects that are studied by at least five students each. Remember, clear formal explanation is important here.

7. Show that the number of odd-order subsets of a set of  $n$  elements is  $2^{n-1}$ .

8. We are organizing a party for small children and need to buy 18 party hats; the store has 5 different designs. In how many ways can we select 18 hats, assuming any hats with the same design are identical and the store has an unlimited number of hats of each design?

9. An accountant returns from vacation with exactly 10 calendar weeks to go before the April 15th deadline to file her clients' income tax statements. She decides to complete at least one tax return per day during the five-day work week, but no more than eight tax returns in any calendar week. Show that there is some set of consecutive work days during which she completes exactly 19 tax returns.