

Department of Mathematics, University of Wisconsin-Madison  
Math 467 — Exam 4 — Fall 2023

NAME : (as it appears on Canvas)

EMAIL: @wisc.edu

PROFESSOR: MIKHAIL IVANOV

**INSTRUCTIONS:**

Time: **50 minutes**

Please write your name on every page.

No Calculators, No Notecards, No Notes

With the exception of the True/False question, and Short Answer question,  
you must justify your claims and use complete sentences in proofs.

You must use correct notation to receive full credit.

You can safely assume that all unknown quantities in this exam,  
e.g.  $a, b, c, n, x, y$ , are always the integers.

Question:	1	2	3	4	5	6	7	Total
Points:	8	8	10	6	6	6	6	50

1. (8 points) For each statement below, choose true or false. You do not need to show your work. Please **fill in** the circle that corresponds to the correct answer.

(a)  $\sum_{d|n} \tau(d) = \sigma(n)$ .

TRUE

FALSE

(b)  $\tau(n) = 5$  if and only if  $n = p^4$ .

TRUE

FALSE

(c) For all positive integer  $n$  holds  $2^{\varphi(n)} \equiv 1 \pmod{n}$ .

TRUE

FALSE

(d) Equation  $\mu(n) = \mu(n+1) = 1$  has solutions in positive integers.

TRUE

FALSE

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

2. (8 points) **On this page, only the answer will be graded. MARK YOUR ANSWER CLEARLY. But you do not need to justify your work.**

(a) Compute  $\tau(120)$ .

(b) Compute  $\sigma(250)$ .

(c) Compute  $\mu(33)$ .

(d) Compute  $\varphi(200)$ .

3. (a) (5 points) Find the last two digits of  $17^{882}$ .

(b) (5 points) Find the last two digits of  $8^{222}$ .

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

4. (6 points) Compute the largest power of 48 that divides  $50!$

5. (6 points) Derive the following expression for the alternating sum of the first  $n \geq 2$  Fibonacci numbers:

$$u_1 - u_2 + u_3 - u_4 + \dots + (-1)^{n+1}u_n = 1 + (-1)^{n+1}u_{n-1}.$$

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

6. (6 points) For the Fibonacci sequence, establish the following:

$$u_{n+5} \equiv 3u_n \pmod{5}.$$

7. (6 points) Let  $t(n) = \tau(n) \cdot \varphi(n)$  and let  $T(n) = \sum_{d|n} t(d)$ . Prove that  $T(n)$  is a multiplicative function.