Department of Mathematics, University of Wisconsin-Madison Math 467 — Exam 4 — Fall 2023

NAME :

(as it appears on Canvas)

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PROFESSOR: MIKHAIL IVANOV

INSTRUCTIONS:

Time: 50 minutes

Please write your name on every page.

No Calculators, No Notecards, No Notes

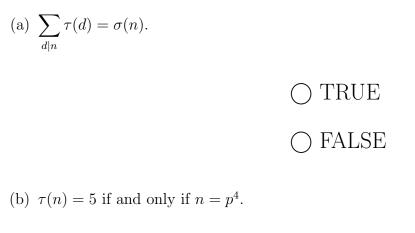
With the exception of the True/False question, and Short Answer question, you must justify your claims and use complete sentences in proofs.

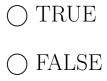
You must use correct notation to receive full credit.

You can safely assume that all unknown quantitites in this exam, e.g. a, b, c, n, x, y, are always the integers.

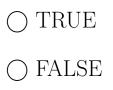
Question:	1	2	3	4	5	6	7	Total
Points:	8	8	10	6	6	6	6	50

1. (8 points) For each statement below, choose true or false. You do not need to show your work. Please fill in the circle that corresponds to the correct answer.





(c) For all positive integer n holds $2^{\varphi(n)} \equiv 1 \pmod{n}$.



(d) Equation $\mu(n) = \mu(n+1) = 1$ has solutions in positive integers.

○ TRUE○ FALSE

- 2. (8 points) On this page, only the answer will be graded. MARK YOUR ANSWER CLEARLY. But you do not need to justify your work.
 - (a) Compute $\tau(120)$.

(b) Compute $\sigma(250)$.

(c) Compute $\mu(33)$.

(d) Compute $\varphi(200)$.

3. (a) (5 points) Find the last two digits of 17^{882} .

(b) (5 points) Find the last two digits of 8^{222} .

First Name: _____

4. (6 points) Compute the largest power of 48 that divides 50!

5. (6 points) Derive the following expression for the alternating sum of the first $n \ge 2$ Fibonacci numbers:

 $u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^{n+1}u_n = 1 + (-1)^{n+1}u_{n-1}.$

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6. (6 points) For the Fibonacci sequence, establish the following:

 $u_{n+5} \equiv 3u_n \pmod{5}.$

7. (6 points) Let $t(n) = \tau(n) \cdot \varphi(n)$ and let $T(n) = \sum_{d|n} t(d)$. Prove that T(n) is a multiplicative function.