Department of Mathematics, University of Wisconsin-Madison

Math 467 — Exam 3 — Fall 2023

NAME :   (as it appears on Canvas)

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INSTRUCTIONS:

Time: **50 minutes**

Please write your name on every page.

No Calculators, No Notecards, No Notes

With the exception of the True/False question, you must justify your claims and use complete sentences in proofs.

You must use correct notation to receive full credit.

You can safely assume that all unknown quantitites in this exam, e.g. a, b, c, n, x, y, are always the integers.

Question:	1	2	3	4	5	6	Total
Points:	8	9	9	8	8	8	50

1. (8 points) For each statement below, CIRCLE true or false. You do not need to show your work.

   (a) $a^{10} \equiv 1 \pmod{11}$ for all values of a.

   TRUE

   FALSE

   (b) The system of linear congruences

   $$\begin{cases} 3x + y \equiv 1 \pmod{5} \\ x + y \equiv 2 \pmod{5} \end{cases}$$

   has at least one solution.

   TRUE

   FALSE

   (c) The equation $3x \equiv 7 \pmod 9$ has NO solutions.

   TRUE

   FALSE

   (d) The equation $7x \equiv 3 \pmod 9$ has NO solutions.

   TRUE

   FALSE

2. (9 points) **On this page, only the answer will be graded. MARK YOUR ANSWER CLEARLY. But you do not need to justify your work.**

   (a) Given that number $\overline{187x2}$ is divisible by 36, find the missing digit x.

   (b) Compute $\tau(30)$.

   (c) Compute $\sigma(30)$.

3. (9 points) Find all primitive Pythagorean triples where $x = 24$. Show your work and write your answer clearly! You do not need to simplify any squares or products. E.g. if you come across something like $16^2 - 3^2$, then you do not need to simplify it.

4. (8 points) Solve

$$\begin{cases} 3x \equiv 4 \pmod{11} \\ x \equiv 2 \pmod{7} \end{cases}$$

5. (8 points) If $\gcd(a, 35) = 1$ show that $a^{12} \equiv 1 \pmod{35}$.

6. (8 points) Find $10! \cdot 20!$ modulo 31.