Department of Mathematics, University of Wisconsin-Madison

Math 467 — Exam 2 — Fall 2023

NAME :   (as it appears on Canvas)

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INSTRUCTIONS:

Time: **50 minutes**

Please write your name on every page.

No Calculators, No Notecards, No Notes

With the exception of the True/False question, you must justify your claims and use complete sentences in proofs.

You must use correct notation to receive full credit.

You can safely assume that all unknown quantitites in this exam, e.g. a, b, c, n, x, y, are always assumed to be the integers.

Question:	1	2	3	4	5	6	7	Total
Points:	8	6	6	6	6	6	12	50

1. (8 points) For each statement below, CIRCLE true or false. You do not need to show your work.

   (a) The product of any two numbers of the form $5n + 3$ is also of the form $5n + 3$.

   <div align="center">TRUE   FALSE</div>

   (b) Let $a, b \in \mathbb{Z}$. If $5a \equiv 5b \pmod{11}$ then $a \equiv b \pmod{11}$.

   <div align="center">TRUE   FALSE</div>

   (c) $1234 \equiv 1 - 2 + 3 - 4 \pmod{11}$.

   <div align="center">TRUE   FALSE</div>

   (d) There are exactly 5 primes between 30 and 50.

   <div align="center">TRUE   FALSE</div>

2. (6 points) Compute the last digit of $(1!)^2 + (3!)^2 + (5!)^2 + (7!)^2 + \ldots$.

3. (6 points) Find some a, b, c such that $ac \equiv bc \pmod{8}$ but $a \not\equiv b \pmod{8}$.

4. (6 points) Find all solutions to the linear equation $5x + 7y = 11$. Express your answer in the form
$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \end{cases} \quad t \in \mathbb{Z}$$
for specific integers x_0, y_0, a, b.

5. (6 points) Prove that $\sqrt{15}$ is not a rational number.

6. (6 points) Formulate and prove divisibility by 5 rule, i.e. how to determine which numbers are divisible by 5 using their decimal representation.

7. (a) (6 points) Find any $k \geq 1$ such that $3^k \equiv 1 \pmod{7}$.

(b) (6 points) Find $0 \leq n \leq 6$ such that $3^{555} \equiv n \pmod{7}$.