Midterm Exam 2

Name:

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- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. **Turn off your phone.**
- Please present your solutions in a clear manner. **Justify your steps.** A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

Problem	Points
1	/17
2	/17
3	/17
4	/17
5	/16
6	/16
Total	/100

1. Suppose that the circumcenter and incenter of $\triangle ABC$ are the same point. Prove that the triangle is equilateral.

2. Prove that the Euler line of the medial triangle of $\triangle ABC$ is the same as the Euler line of $\triangle ABC$.

3. Show that for any triangle with inradius r, circumradius R and exradii r_a, r_b, r_c holds

$$r_a + r_b + r_c = 4R + r.$$

4. Given trapezoid with bases 12 and 16 and legs 7 and 9. Find angles of the trapezoid.

5. Lines PA and PB are tangent to a circle centered at O; let A and B be the tangent points. A third tangent to the circle is drawn; it intersects with segments PA and PB at points X and Y, respectively. Prove that the value of angle XOY does not depend on the choice of the third tangent.

6. Let I_a, I_b, I_c denote the centers of the exscribed circles correspond ing to vertices A, B and C respectively for $\triangle ABC$. Let X, Y, Z denote the points where the incircle of $\triangle ABC$ meets the three sides of $\triangle ABC$. Prove that $\triangle XYZ \sim \triangle I_a I_b I_c$.

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