Final Exam

Name:

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- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. **Turn off your phone.**
- Please present your solutions in a clear manner. **Justify your steps.** A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

Problem	Points
1	/12
2	/12
3	/12
4	/12
5	/16
6	/12
7	/12
8	/12
Total	/100

1. Recall that the Gergonne point of a triangle is the point of intersection of the three segments joining the vertices of the triangle with points of tangency of the incircle with the opposite side. Let T be the Gergonne point of $\triangle ABC$. Show that if T coincides with the **incenter** of $\triangle ABC$, then the triangle must be equilateral.

2. In quadrilateral $ABCD$, suppose that $AB \parallel CD$ and $\angle B = \angle D$. Show that $ABCD$ is a parallelogram.

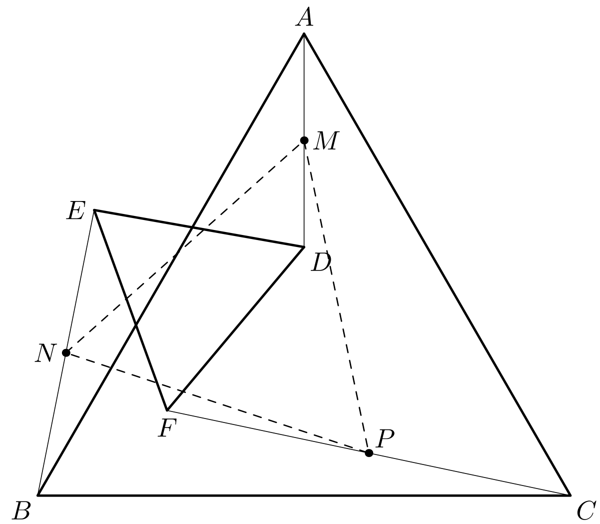
3. Find angle between vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\left(\vec{a} - \vec{b}\right)^2 + \left(2\vec{a} - \vec{b}\right)^2 = 56$.

4. Show that cevians bisecting perimeter of the triangle are concurrent.

5. Let M be a point outside of circle ω with center O. The line OM intersects circle ω at points A and B with $MA = a$, $MB = b$. Tangent line from M touch ω at point C. Point H is the projection of point C on AB. Perpendicular line from O to AB intersects ω at P. Find lengths MO, MP, MC and MH.

6. Let ω be a circle, having an arm of right angled triangle as a diameter. The circle cut the hypotenuse at ratio $1 : 3$. Find angles of the right angled triangle.

7. Given two equilateral triangles $\triangle ABC$ and $\triangle DEF$ and points M, N, P: $MA = MD$, $NB = NE$ and $PC = PF$. Prove that $\triangle MNP$ is an equilateral triangle.



8. In the triangle ABC, let D, E and F are points on sides BC, AC, and AB, respectively. Let $AF/FB = r$, $BD/DC = s$, and $CE/EA = t$. Let X be the intersection of AD and BE, Y be the intersection of BE and CF, and Z be the intersection of CF and AD. Show, that

$$\frac{\mathcal{A}_{XYZ}}{\mathcal{A}_{ABC}} = \frac{(rst-1)^2}{(rs+r+1)(st+s+1)(tr+t+1)}.$$

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