Math/Stat 431: Introduction to the Theory of Probability lectures 1 and 4, Fall 2024

Course Details

Official Course Description: Topics covered include axioms of probability, random variables, the most important discrete and continuous probability distributions, expectation and variance, moment generating functions, conditional probability and conditional expectations, multivariate distributions, Markov's and Chebyshev's inequalities, laws of large numbers, and the central limit theorem.

Instructional Modality: Classroom Instruction.

Meeting Time and Location: Sewell Social Sciences Hall, room 6240.

lecture 004	MWF: 13:20PM–14:10PM
lecture 001	MWF: 14:25PM–15:15PM

Instructor: Mikhail Ivanov, Teaching Faculty *Email:* mivanov@wisc.edu *Office:* Van Vleck B127

Instructor Office hours: WF: 3:30PM–4:20PM in Van Vleck Hall, room B224 (please check Canvas page for updates), or by Appointment.

Grader and/or Course Assistant: We will have a grader and a course assistant that has yet to be determined. The Syllabus and Canvas page will be updated once they are assigned.

Credits hours: 3.

How Credit Hours are Met by the Course. This course meets the Traditional Carnegie Definition for how credit hours are met by the course. Students in the course have 2.5 hours/week of direct faculty instruction during class time and are expected to work on course learning activities (reading, writing, problem sets, studying, etc) for a minimum of 2 hours outside of classroom per course credit (i.e. 6 hours/week). The syllabus includes more information about meeting times and expectations for student work.

Course Designations and Attributes:
Breadth – Natural Science
Level – Advanced
L&S Credit – Counts as Liberal Arts and Science credit in L&S

Requisites: MATH 234 or 376 or graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program

Course Learning Outcomes

By the conclusion of this course, students should have a thorough understanding of:

- Recall and state the formal definitions of the mathematical objects and their properties used in probability theory (e.g., probability spaces, random variables and random vectors and their probability distributions, named distributions, conditional probability, independence, linearity of expectation, etc.).
- Use such definitions to argue that a mathematical object does or does not have the condition of being a particular type or having a particular property (e.g., whether certain events or random variables are independent or not, whether a random variable has one of the named distributions, whether or not a sequence of random variables is exchangeable, etc.).
- Recall and state the standard theorems of probability theory. (e.g., Bayes' theorem, the law of large numbers, the central limit theorem, etc.), and apply these theorems to solve problems in probability theory.
- Use multiple approaches to compute and estimate probabilities and expectations (e.g., using the indicator method, using conditioning, estimating probabilities using normal or Poisson approximation etc.).
- Construct mathematical arguments related to the above definitions, properties, and theorems, including the construction of examples and counterexamples.
- Convey his or her arguments in oral and written forms using English and appropriate mathematical terminology and notation (and grammar).
- Model simple real-life situations using techniques in probability theory and calculate probabilities and expectations associated with those models.

Course Overview

Math 431 is an introduction to the theory of probability, the part of mathematics that studies random phenomena. We model simple random experiments mathematically and learn techniques for studying these models. Topics covered include axioms of probability, random variables, the most important discrete and continuous probability distributions, expectations, moment generating functions, conditional probability and conditional expectations, multivariate distributions, Markov's and Chebyshev's inequalities, laws of large numbers, and the central limit theorem.

Tentative Course Schedule

Below is a tentative course schedule. There could be a slight change depending on our progress.

Week	M	W	F	Topics	Sections
Week 1		09/04	09/06	Axioms of probability, sampling, review of counting, infinitely many outcomes	1.1-1.3
Week 2	09/09	09/11	09/13	Consequences of the rules of probability, random variables	1.4-1.5
Week 3	09/16	09/18	09/20	Conditional probability, Bayes formula, independence	2.1-2.3

Week 4	09/23	09/25	09/27	Independent trials, named distributions, conditional independence	2.4-2.5
Week 5	09/30	10/02	10/04	Probability distribution of a random variable, expectation and variance	3.1-3.3
Week 6	10/07	10/09	10/11	Gaussian distribution, normal approximation for the binomial distribution	3.3-3.5
Week 7	10/14	10/16	10/18	Normal approximation, law of large numbers, confidence intervals, the Poisson distribution	4.1-4.4
Week 8	10/21	10/23	10/25	Poisson approximation, exponential distribution, moment generating function	4.5, 5.1-5.2
Week 9	10/28	10/30	11/01	Joint distribution of random variables, the joint pmf, the joint pdf,	6.1-6.2, 6.4
Week 10	11/04	11/06	11/08	Joint distributions of continuous random variables, Sums of independent random variables,	6.3, 7.1
Week 11	11/11	11/13	11/15	Sums of independent random variables cont., expectations of sums and products	7.2, 8.1-8.2
Week 12	11/18	11/20	11/22	Expectation and variance of the sample mean, coupon collector,	8.2-8.4
Week 13	11/25	09/11	09/13	Covariance and correlation, law of large numbers, central limit theorem	8.4, 9.1-9.3
Week 14	12/02	09/11	09/13	Conditional distribution and conditional expectation	10.1-10.3
Week 15	12/09	12/11		Further conditional expectation examples, review	10.3-10.4

Textbook, Course Website, Digital Tools and Other Course Materials

- Required textbook: Anderson, Seppäläinen, Valkó: Introduction to Probability, Cambridge University Press, 2017
- `Canvas` (https://canvas.wisc.edu/courses/417744) is our Learning Management System. All important course information will be relayed through Canvas. Make sure your Canvas notification setting for "Announcement" is "Notify immediately" so that you receive messages promptly. Notifications settings can be accessed here: https://canvas.wisc.edu/profile/communication.
- `Zoom`(https://uwmadison.zoom.us) will be used for remote office hours.
- `Gradescope`(https://www.gradescope.com/) will be used to grade homework and exams. Instructions will be shared later.
- `Piazza` (https://piazza.com/class/m0fo51gl18m4kq), the online discussion forum.

Class Components

Lectures. Lectures will be delivered in-person and synchronously three times a week during regular class hours. Lectures will involve some active learning components (e.g., problems and small group work). Attendance and active engagement is expected.

Piazza. The online discussion forum, known as `Piazza`, will be used to discuss ideas or questions outside of class. Piazza can be used to get hints on homework problems, but no one is allowed to post entire solutions to homework assignments. However, feel free to post entire solutions to practice problems. Logistic questions are usually better posted on Piazza rather than emailed to the instructor, so all students can benefit from the answer. (e.g., exam dates and locations). Please do not use email for math questions.

You can use private question in Piazza to communicate with Instructor about personal circumstances.

Any posts containing comments (either positive or negative) about the instructors, the class, the students, or anything else, will be deleted. Unprofessional conduct may result in disciplinary action.

Homework. Assignments will be posted on Canvas, and they will have to be submitted in Gradescope as a single pdf file. Written assignments will be several questions long and will be assigned weekly usually due roughly every Wednesday. Please make sure all pages are in the right orientation when you convert them. Do not hand in your rough draft or first attempt. Papers that are unreadable or disorganized cannot be graded. It is a good habit to download your submission every time and check everything is fine.

Collaborating with other students on the homework is encouraged, but you must write up all reasoning and solutions on your own (in other words, no copying). Failure to abide by this guideline could be construed as a form of academic dishonesty.

Each problem should be completed with neat, understandable, detailed solutions and explanations. Your explanations and proofs must be sound and rigorous, paying attention to detail and clarity. Computations without appropriate explanation will not receive credit, even if the final numerical answer is correct.

Late homework will generally not be accepted. Since it is quite likely that during the semester you will either experience a technical difficulty (e.g., missed the deadline, your computer shut down as you were submitting it, internet outage, etc) or a personal emergency (being sick, attending a funeral, etc), the two lowest HW scores will be dropped. You do not need to contact your instructor if such a situation does come up.

Homework quizzes. There will be up to 14 short quizzes (roughly 1 per week), usually due on Sunday. Quizzes will be short assignments on Canvas to primarily test comprehension of lectures and prerequisite material. A quiz could involve definitions, computation, or comprehension. Late quizzes will not be accepted. Two lowest Homework quizzes scores will be dropped.

In-class Quizzes. To help prepare for the exams we will have short in-class "surprise" quizzes (on Wednesdays). It is your responsibility to attend lectures and quizzes. The lowest score will be dropped.

Exams. The course will have three exams. We will have two evening Midterm Exams and a Final Exam, places to be scheduled by the University. Final Exam will be cumulative.

Midterm Exam 1	Wednesday, October 9	07:30pm–09:00pm
Midterm Exam 2	Wednesday, November 13	07:30pm–09:00pm
Final Exam	Wednesday, December 18	07:25pm–09:25pm

Students with academic or religious conflict with one of the exams should notify the instructor as soon as possible, and no later than the third week of the semester.

Calculator Policy. During an exam no books, notes, calculators, cell phones, pagers, or any electronic devices will be allowed.

Grading

In this course, you will be evaluated based on components described above with their corresponding percentages:

Homework	15%
Homework quizzes	3%
In-class quizzes	2%
Midterm Exam 1	25%
Midterm Exam 2	25%
Final Exam	30%

Grading Scale. The following grade lines are guaranteed in advance. A percentage score in the indicated range guarantees at least the letter grade next to it.

$$\text{A} \geq 90\% > \text{AB} \geq 88\% > \text{B} \geq 78\% > \text{BC} \geq 76\% > \text{C} \geq 66\% > \text{D} \geq 50\% > \text{F}$$

Final letter grades are not curved but the grade lines above may be lowered at the end. Class attendance is not part of the grading.

How to Succeed in This Course

Here are a couple of suggestions for being successful in this class:

- The best way to learn math is by doing it. Try to work through the examples in the textbook before reading them, and try to solve as many practice problems as you can (on top of the homework problems). Let me know if you run out of problems to solve!
- Attend the lectures, and try to be active in class. Ask questions if something is not clear.
- Read the textbook.
- In general, try to keep up with the material. We cover several topics in the class that build on each other, and it will be hard to catch up if you get behind in the material.
- Use Piazza to ask questions related to the course material, and try to answer questions of other students if you can.
- Take advantage of the office hours! I am happy to help you, but I cannot do that if you do not ask for it.
- The Math Learning Center (in particular the Course Assistant and the Proof Table) could be a useful resource.

Campus Resources for Academic Success

- University Health Services
- Undergraduate Academic Advising and Career Services
- Office of the Registrar
- Office of Student Financial Aid
- Office of Student Assistance and Support
- Graduate Student Services

Academic Policies and Statements

- Academic Calendar and Religious Observances
- Academic Integrity Statement
- Accommodations for Students with Disabilities
- Course Evaluations
- Diversity and Inclusion Statement
- Mental Health and Well-Being Statement
- Privacy of Student Records and the Use of Audio Recorded Lectures Statement
- Students' Rules, Rights and Responsibilities
- Teaching and Learning Data Transparency Statement

Mathematica

Mathematica and Wolfram—Alpha Pro are available at no charge to UW Madison students. They are useful for: (a) solving problems, (b) obtaining step-by-step solutions, and (c) writing programs with the assistance of Artificial Intelligence. (AI chat is built in.) To get access, go to www.wolfram.com/siteinfo and enter your University of Wisconsin email. (Here are click-by-click steps if you need them: wolfr.am/UWMadison) Learn how to use Mathematica at www.wolfram.com/wolfram-u/