Department of Mathematics, University of Wisconsin-Madison

Math 431 — Midterm Exam 1 — Spring 2025

NAME : (as it appears on Canvas)

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PROFESSOR: Mikhail Feldman or Mikhail Ivanov

INSTRUCTIONS:

Time: **90 minutes**

- This exam contains 6 questions some with multiple parts, 10 pages (including the cover) for the total of 83 points. Read the problems carefully and budget your time wisely.
- You are allowed a single sheet of hand-written notes.
- **NO CALCULATORS** or other electronic devices are to be used. Turn off your phone so as to not disturb others.
- You **do not** need to simplify binomial coefficients or factorials, except when you are asked to do it.
- Please present your solutions in a clear manner. Cross out any writing that you do not wish to be graded.
- Justify your steps.
- If you use an additional page for a particular problem, be sure to **CLEARLY** indicate this on the problem's page so I know to look further.

Question:	1	2	3	4	5	6	Total
Points:	16	12	8	18	21	8	83
Score:							

1. (16 points) The following questions are **Answers Only**.

   (a) In a five-card hand drawn at random from a well-shuffled standard deck, find the probability that the hand has exactly three aces.

   (b) Let X and Y be random variables taking only values 1 and 2. Give the precise conditions for X and Y to be independent.

   (c) We sample two numbers without replacement with order from the set $\{1, 2, 3, 4, 5, 6\}$. Describe the sample space of the experiment.

   (d) Suppose that A and B are events with $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, and $P(A \cap B) = \frac{1}{4}$. Find the probability of $A^c \cup B$.

2. A certain candy company put golden tickets in the wrappers of 2% of their candy bars. If you find a golden ticket, you win a trip to their candy factory. Assume that every candy bar is equally likely to have a golden ticket.

   (a) (6 points) Suppose you bought 15 candy bars. What is the probability you find exactly 3 tickets?

   (b) (6 points) Suppose you buy candy bars until you find your first ticket. What is the probability that you must buy more than 50?

3. (8 points) A woman is pregnant with twin boys. Twins may be either identical or fraternal. Suppose that 1/3 of twins born are identical, that identical twins have a 50% chance of being both boys and a 50% chance of being both girls, and that for fraternal twins each twin independently has a 50% chance of being a boy and a 50% chance of being a girl. Given the above information, what is the probability that the woman's twins are identical?

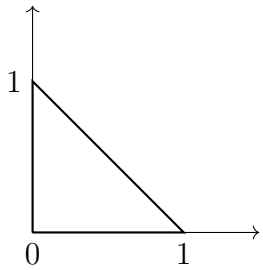
4. A box contains 8 green balls, 6 red balls, and 4 yellow balls. Two balls are sampled uniformly at random without replacement.

   (a) (6 points) What is the probability that one ball is green and another is yellow? (First write down your sample space.)

   (b) (6 points) What is the probability that at least one of the balls is red?

(c) (6 points) Let X be the number of red balls in the sample. Find CDF of X and draw the graph of this function.

5. We choose a point (X, Y) uniformly from the triangle with vertices $(0, 0)$, $(0, 1)$ and $(1, 0)$. Let $Z = 2X - 1$.



(a) (4 points) Find the range of the random variable Z.

(b) (5 points) Compute $P(Z > \frac{2}{3})$.

(c) (6 points) Compute the cumulative distribution function of Z.

(d) (6 points) Find the probability density function of Z.

6. (8 points) Professor May B. Right often has her facts wrong, and answers each of her students' questions incorrectly with probability 1/4, independent of other questions. In each lecture, May is asked 0, 1, or 2 questions with equal probability 1/3. Find the probability that she gave at least one wrong answer during one lecture.

SCRATCH PAPER - DO NOT REMOVE FROM YOUR EXAM.
SCRATCH WORK WILL NOT BE GRADED