Midterm 2

Name:

Student ID number:

- This is exam for section 002, instructor Mikhail Ivanov.
- **There are 7 problems on the exam, some of them have multiple parts.**
- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. **Turn off your phone.**
- You do not have to carry out complicated numerical computations, but you should simplify your answer if it is possible with reasonable effort. In particular, geometric series must be evaluated.
- Please present your solutions in a clear manner. **Justify your steps.** A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.
- You can use the attached table for the values of the $\Phi(x)$ function.
- **Important note:** there are two problems where you will have to use normal or Poisson approximation to estimate a certain probability. (You will have to decide which one to use.) If you use the normal approximation then you should not worry about the continuity correction, but you should compute the numerical answer using the attached table. (The numbers are set up so that you do not need a calculator to compute the answer.) If you use the Poisson approximation then your final answer should be in the form that is easily computable with a simple calculator.

Problem	Points
1	/20
2	/10
3	/10
4	/20
5	/20
6	/10
7	/10
Total	/100

1. Assume everyone in Madison owns between zero and three cats and between zero and two dogs. Let X be the number of cats (Y be the number of dogs) a randomly chosen person owns. Suppose the joint pmf of X and Y is given by

		Cats			
		0	1	2	3
	0	12/48	6/48	3/48	3/48
Dogs	1	8/48	4/48	2/48	2/48
	2	4/48	2/48	1/48	1/48

(a) Find marginal distribution of X and Y.

(b) Are X and Y independent? Justify your answer mathematically.

(c) Let $Z = X + Y$ be the totally number of pets (cats and dogs) the randomly chosen person owns. Compute $E(Z)$.

2. Let X be a random variable with moment generating function $M(t) = \left(1 - \frac{t}{2}\right)^{-2}$ when $|t| < 2$. What is the variance of X?

3. The owner of a certain website is studying the distribution of the number of visitors to the site. Every day, a million people independently decide whether to visit the site, with probability $p = 2 \times 10^{-6}$ of visiting. Give a good approximation for the probability of getting at least three visitors on a particular day.

4. Sides of fair dice are 2, 2, 3, 4, 5, 6. Roll the dice four times.

   (a) Find the probability that you roll 2 once, 3 once, and 4 twice (at some order). Simplify your answer.

   (b) Find the probability that you roll 6 exactly once.

   (c) Approximate the probability that, after 450 rolls, you have rolled 130 or more 2's.

5. Let X and Y be random variables with joint density

$$f(x,y) = \begin{cases} cx & (x,y) \in [0,1] \times [0,2] \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c.

(b) Find the marginal densities for X and Y.

(c) Find $P(X > Y)$.

6. Suppose that X has probability density function given by $\frac{1}{x^2}$ for $x > 1$ and 0 otherwise. Find the pdf of $|X - 3|$.

7. Assume a baseball team wins any given baseball game with probability p (regardless of who there opponent is). Suppose they played 100 games and won 60 of them. How confident are you that the true probability of winning a game is within 0.1 of the observed probability of winning a game $\hat{p} = 0.6$?

Table of values for $\Phi(x)$, the cumulative distribution function of a standard normal random variable

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998