## Midterm 1

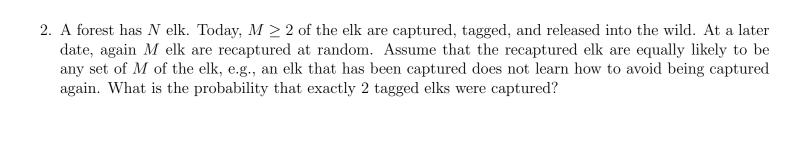
## Name:

## Student ID number:

- This is exam for section 002, instructor Mikhail Ivanov.
- There are 6 problems on the exam, some of them have multiple parts.
- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. Turn off your phone.
- You do not have to carry out complicated numerical computations, but you should simplify your answer if it is possible with reasonable effort. In particular, geometric series must be evaluated.
- Please present your solutions in a clear manner. Justify your steps. A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

Problem	Points
1	/15
2	/15
3	/15
4	/15
5	/25
6	/15
Total	/100

- 1. You have a 6-sided, a 10-sided, and a 12-sided die. You choose one at random and roll it.
  - (a) What is the probability that your roll is greater than or equal to 6?
  - (b) Given that your roll is greater than or equal to 6, what is the probability you rolled the the 10-sided?



- 3. A certain candy company put golden tickets in the wrappers of 1% of their candy bars. If you find a golden ticket you win a trip to their candy factory. Assume that every candy bar is equally likely to have a golden ticket.
  - (a) Suppose you bought 10 candy bars. What is the probability you find exactly 1 ticket? exactly 2 tickets?
  - (b) Suppose you bought candy bars until you find your first ticket. What is the probability that you must bought more than 100?

- 4. All the buildings in a certain city have between 1 and 10 floors. Strangely the city has k buildings with k floors for each  $k = 1, \ldots, 10$ . (That is, there is 1 building with a single story, 2 buildings two stories tall, etc.)
  - (a) Suppose pick a building uniformly at random to enter. Let X be the number of floors the building has. Write down the probability mass function for X.
  - (b) What is the probability that the building has more than 5 floors?
  - (c) Suppose you start climbing the stairs and reach the second floor before you get tired and stop. Now what is the probability that the building has more than 5 floors?

(You might find the identity  $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$  useful.)

- 5. Pick a uniformly chosen random point inside the triangle with vertices (0, 0), (10, 0) and (10, 10). (Draw a picture!) Let X be the x-coordinate of the chosen point.
  - (a) What is the probability P(5 < X < 8)?
  - (b) What is the c.d.f. of X?
  - (c) What is the p.d.f. of X?
  - (d) What is the  $\mathbb{E}[X]$ ?
  - (e) Now we pick two uniformly chosen random points inside the triangle. Let  $X_1$  and  $X_2$  be the x-coordinates of the chosen points. What is the probability  $P(\min(X_1, X_2) \ge 4)$ ?

6. Students in a school have a choice of three foreign language courses (F)rench, (G)erman and (S)panish (students can attend more than one course or none of them). Let F, G, S denote the event that a randomly selected student plans to attend respectively French, German and Spanish courses respectively. From past experience suppose the school knows that the probabilities of these events are

$$P(F) = 0.6,$$
  $P(G) = 0.6,$   $P(S) = 0.5.$ 

and further

$$P(F \cap G) = 0.4,$$
  $P(F \cap S) = 0.3,$   $P(G \cap S) = 0.2$ 

and finally

$$P(F \cap G \cap S) = 0.1.$$

- (a) Find the probability that a randomly selected student plans to attend at least one of the three language courses.
- (b) Find the probability that a randomly selected student plans to attend **only** Spanish.

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