Midterm 1

Name:

Student ID number:

- This is exam for section 002, instructor Mikhail Ivanov.
- **There are 6 problems on the exam, some of them have multiple parts.**
- Read the problems carefully and budget your time wisely.
- No calculators or other electronic devices, please. **Turn off your phone.**
- You do not have to carry out complicated numerical computations, but you should simplify your answer if it is possible with reasonable effort. In particular, geometric series must be evaluated.
- Please present your solutions in a clear manner. Justify your steps. A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

Problem	Points
1	/15
2	/15
3	/15
4	/15
5	/25
6	/15
Total	/100

1. You have a 6-sided, a 10-sided, and a 12-sided die. You choose one at random and roll it.
   (a) What is the probability that your roll is greater than or equal to 6?
   (b) Given that your roll is greater than or equal to 6, what is the probability you rolled the the 10-sided?

2. A forest has N elk. Today, $M \geq 2$ of the elk are captured, tagged, and released into the wild. At a later date, again M elk are recaptured at random. Assume that the recaptured elk are equally likely to be any set of M of the elk, e.g., an elk that has been captured does not learn how to avoid being captured again. What is the probability that exactly 2 tagged elks were captured?

3. A certain candy company put golden tickets in the wrappers of 1% of their candy bars. If you find a golden ticket you win a trip to their candy factory. Assume that every candy bar is equally likely to have a golden ticket.

   (a) Suppose you bought 10 candy bars. What is the probability you find exactly 1 ticket? exactly 2 tickets?

   (b) Suppose you bought candy bars until you find your first ticket. What is the probability that you must bought more than 100?

4. All the buildings in a certain city have between 1 and 10 floors. Strangely the city has k buildings with k floors for each $k = 1, \ldots, 10$. (That is, there is 1 building with a single story, 2 buildings two stories tall, etc.)

   (a) Suppose pick a building uniformly at random to enter. Let X be the number of floors the building has. Write down the probability mass function for X.

   (b) What is the probability that the building has more than 5 floors?

   (c) Suppose you start climbing the stairs and reach the second floor before you get tired and stop. Now what is the probability that the building has more than 5 floors?

   (You might find the identity $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ useful.)

5. Pick a uniformly chosen random point inside the triangle with vertices (0, 0), (10, 0) and (10, 10). (Draw a picture!) Let X be the x-coordinate of the chosen point.

   (a) What is the probability $P(5 < X < 8)$?

   (b) What is the c.d.f. of X?

   (c) What is the p.d.f. of X?

   (d) What is the $\mathbb{E}[X]$?

   (e) Now we pick two uniformly chosen random points inside the triangle. Let X_1 and X_2 be the x-coordinates of the chosen points. What is the probability $P(\min(X_1, X_2) \geq 4)$?

6. Students in a school have a choice of three foreign language courses (F)rench, (G)erman and (S)panish (students can attend more than one course or none of them). Let F, G, S denote the event that a randomly selected student plans to attend respectively French, German and Spanish courses respectively. From past experience suppose the school knows that the probabilities of these events are

$$P(F) = 0.6, \qquad P(G) = 0.6, \qquad P(S) = 0.5.$$

and further

$$P(F \cap G) = 0.4, \qquad P(F \cap S) = 0.3, \qquad P(G \cap S) = 0.2$$

and finally

$$P(F \cap G \cap S) = 0.1.$$

(a) Find the probability that a randomly selected student plans to attend at least one of the three language courses.

(b) Find the probability that a randomly selected student plans to attend **only** Spanish.

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