COMPACT SUBSETS OF THE BAIRE SPACE
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Let ω^ω be the Baire space, infinite sequences of natural numbers with the product topology. In this topology a set $K \subset \omega^\omega$ is compact iff there exists a finite branching tree $T \subseteq \omega^{<\omega}$ such that

$$K = [T] = \{x \in \omega^\omega : \forall n \in \omega \; x \upharpoonright n \in T\}.$$

Theorem 1 *If there exists a countable standard model of ZFC, then there exists M, a countable standard model of ZFC, $N \supseteq M$, a generic extension of M, and $T \in N$ a finite branching subtree of $\omega^{<\omega}$ with the properties that*

1. $\forall f \in [T] \cap N \; \exists g \in M \cap \omega^\omega \; f(n) < g(n)$ *for all but finitely many $n \in \omega$ and*
2. $\forall g \in M \cap \omega^\omega \; \exists f \in [T] \cap N \; g(n) < f(n)$ *for infinitely many $n \in \omega$.*

This is related to Michael's problem [3] of whether there must be a Lindelöf space X such that $X \times \omega^\omega$ is not Lindelöf and M.E.Rudin's characterization of that problem [4], see also Alster [1] and Lawrence [2].

References

[1] K.Alster, On the product of a Lindelöf space with the space of irrationals under Martin's Axiom, Proceedings of the American Mathematical Society, 110(1990), 543-547.

[2] B.Lawrence, The influence of a small cardinal on the product of a Lindelöf space and the irrational, Proceedings of the American Mathematical Society, 110(1990), 535-542.

[3] E.Michael, The product of a normal space and a metric space need not be normal, Bulletin of the American Mathematical Society, 69(1963), 375-376.

[4] M.E.Rudin, An analysis of Michael's problem, preprint 1993.