Errata for "The Mathematics of Shock Reflection-Diffraction and von Neumann's Conjectures" (2018) by G.-Q. G. Chen and M. Feldman.

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CHAPTER 1

Page 32, line 11: "in the sense of Definition 2.3.3" should be "in the sense of Definition 2.3.1";

Page 34, line 8 in Theorem 2.6.7: "in the sense of Definition 2.3.3" should be "in the sense of Definition 2.3.1";

CHAPTER 4

page 147, line 14 (displayed equation): the $\frac{\pi}{2}$ should be π, i.e. the formula becomes:

$$\theta_0 \leq \theta_k \leq \pi - \theta_0.$$

Page 161, line -9: in "... $\leq \|f\|^{(-1-\alpha),\{0\}}_{2,\alpha,(0,s)} \leq M_{\rm ob}$", f should be changed to $f_{\rm ob}$.

Page 166. In Lemma 4.5.8 add assumption $g_3 = 0$. Note that the proof is given for that case. All applications of this lemma in the book are for the case $g_3 = 0$.

Pages 167-169. Corrections for proof of Lemma 4.5.8: see at the end of these notes.

Page 170. In Lemma 4.5.9 add assumption $g_3 = 0$ (compare with correction for Lemma 4.5.8 above). For the applications of Lemma 4.5.9, see correction to Proposition 4.8.4 below.

Page 199, line -11: in $\lambda^2\sqrt{\lambda^2+1} > \frac{\lambda}{2}$, the inequality sign have to be changed, i.e. should be $\lambda^2\sqrt{\lambda^2+1} < \frac{\lambda}{2}$.

Page 199, line -5: Problem (4.8.8) should be changed to Problem (4.7.2).

Page 206. In equations (4.8.5) and (4.8.6), in the term $\sum_{j=1,3} \|g_j\|^{(-\alpha),\partial\Gamma_j}_{1,\alpha,\Gamma_1}$, the Γ_1 in the subscript should be Γ_j.

Page 209. In Proposition 4.8.4 we should assume $g_3 = 0$. This correction justifies the use of Lemma 4.5.9 in the proof. For the applications of Proposition 4.8.4, see correction to the proof of Proposition 4.8.7 below.

Page 215. In the proof of Proposition 4.8.7, the following corrections are needed, related to the requirement $g_3 = 0$ in Proposition 4.8.4 (introduced in the correction above): Spaces $\mathcal{C}_D$ and $\mathcal{C}_T$ are now defined as:

$$\mathcal{C}_D = \Big\{u \in C^{(-1-\alpha_1),\{P_1\}\cup\overline{\Gamma_2}}_{2,\alpha_1,\Omega} \;:\; \mathbf{b}^{(3)}(\mathbf{x})\cdot Du = 0 \;\text{ on } \Gamma_3,\; u|_{P_0} = 0\Big\},$$

$$\mathcal{C}_T = \left\{\begin{array}{l}(f, g_1, g_2, g_3) \in C^{(1-\alpha_1),\{P_1\}\cup\overline{\Gamma_2}}_{0,\alpha_1,\Omega} \times C^{(-\alpha_1),\partial\Gamma_1}_{1,\alpha_1,\Gamma_1} \times C^{\alpha_1}(\overline{\Gamma_2}) \times C^{(-\alpha_1),\partial\Gamma_3}_{1,\alpha_1,\Gamma_3} \\ \text{with } \; g_3 \equiv 0 \;\text{ on } \Gamma_3\end{array}\right\}.$$

CHAPTER 5

Page 218. The title of §5.1.3 should be "Tangential derivative of the Rankine-Hugoniot conditions on Γ_{shock}".

Page 219, line -5: in first bracket, the sign should be changed from minus to plus for $\rho_1\tau$. This term however vanishes since it is multiplied by the orthogonal vector $\tilde{\nu}$, so this typo does not affect the rest of the calculation.

CHAPTER 8

Page 282. In (8.1.3), the $C^1(\overline{\Lambda} \setminus \overline{P_0P_1P_2})$ should be $C^1(\overline{\Lambda} \setminus (S_0 \cup \overline{P_0P_1P_2}))$, i.e. $D\varphi$ has also discontinuity across incident shock S_0 (in addition to the reflected shock $P_0P_1P_2$)

CHAPTER 11

Page 437 line 2: $\partial\Omega^{(\infty)}$ to be changed to $\partial\Omega^{(i_j)}$

CHAPTER 12

Page 510. In (12.7.16), typo: in superscript, should be $\overline{\Gamma_{sym}}$ instead of $\overline{\partial_{sym}Q^{iter}}$.

Page 516, displayed equation before (12.7.26). Typo: first term on the rhs: $\hat{u}(u(\mathfrak{F}^{-1}(\xi_1, \xi_2)))$ should be changed to to $\hat{u}(\mathfrak{F}^{-1}(\xi_1, \xi_2))$.

CHAPTER 13

Page 535, line -4: "By (13.4.11)" should be changed to "By Rankine-Hugoniot condition for $\varphi^{\pm}$ on S".

Page 572, Lemma 13.9.6 (iii): In second displayed equation, change $C^{2,\beta}(\Omega^a_{(1+\frac{1}{2}\kappa)g})(b_1, b_2)$ to $C^{2,\beta}(\overline{\Omega^a_{(1+\frac{1}{2}\kappa)g}(b_1, b_2)})$

CHAPTER 15

(15.1.2), discontinuity across S_0 should be added to $C^1(\overline{\Lambda} \setminus \overline{\Gamma_{\text{shock}}})$, to have $C^1(\overline{\Lambda} \setminus (S_0 \cup \overline{\Gamma_{\text{shock}}}))$. This is similar to correction in (8.1.3)

In (15.1.3) $D\varphi(P_0)$ should be changed to $D\varphi_{|\Omega}(P_0)$ to clarify, since $D\varphi$ is discontiuous across Γ_{shock} at P_0.

CHAPTER 16

Page 680: in (16.6.59), in the expression for $a_{22}(\mathbf{p}, z, \mathbf{y})$, the $\frac{p_2}{p_1}$ should be changed to $\frac{1}{p_1}$.

Page 687, first line of Section 16.6.5: P_0 should be changed to P_1.

Page 689. A new Section 16.7 needs to be added to extend the results of Section 11.6 to all (supersonic and subsonic) regular reflections. These results, given below, are used in the proof of Lemma 17.10.2 (which extends to all types of wedge angles the proof of Lemma 13.5.1 where the results of Section 11.6 are used). See the pages at the end of these notes for the added Section 16.7,

CHAPTER 17

Page 710, line 6: After this line, add the new item:
(e) $N_4 = 10C$, where C is from Corollary 9.1.3, extended by Proposition 15.2.2.
Page 728, line 8. Words "property (i)" should be "property (i) of Lemma 12.4.2, which holds in the present case by Lemma 17.3.17 ". Also, on line (10), the "(i)" should be changed to "(i) of Lemma 12.4.2".

Page 732, lines 2 and 18: "Lemma 16.4.2" should be "Lemma 16.4.1".

See next pages for:
- corrections to proof of Lemma 4.5.8.
- new added Section 16.7.

Corrections for proof of Lemma 4.5.8.

We first note that in (4.5.126) the property $\hat{b}_1^{(1)} \le -\lambda$ holds only on $\Gamma_1 \cap \{x_1 < \varepsilon/2\}$, not on the whole Γ_1, as follows from (4.5.102) and (4.5.123). This typo/mistake requires the following updates in the remainder of proof, i.e. in the argument on pp. 168-169. Constants C, C_k below depend only on λ, ε, M.

First we show (4.5.119). We use $w(x_1)$ defined by (4.5.128) with $\hat{M} = 2\frac{M}{\lambda\varepsilon}$ and large $C_1 > 0$ defined below. Then $w(0) = 0$,

$$\frac{\lambda\varepsilon}{2} w'' + Mw' + M = 0.$$

Also, (4.5.126) holds, and from this and $w(0) = 0$, it follows that $w(x_1) > 0$ for $x_1 > 0$. Let

$$V(x_1, x_2) = w(x_1) + C_2,$$

where large $C_2 > 0$ will be fixed below. Then the argument on pp. 168-169 shows that $\mathcal{L}(V) - f < 0$ in Ω and $\mathcal{B}^{(k)}V \le g_k$ on Γ_k for $k = 2, 3$ if C_1 is large, where $g_3 = 0$. Note that in showing $\mathcal{L}(V) - f < 0$, we consider separately the cases $\Omega \cap \{0 < x_1 < \varepsilon/2\}$ and $\Omega \cap \{x_1 \ge \varepsilon/2\}$ (not ε as on p. 168). Also, $V \ge C_2 > 0$ in $\overline{\Omega}$, thus on Γ_0. It remains show that $\hat{\mathcal{B}}^{(1)}V \le g_1$ on Γ_k. We use that $\hat{b}_0^{(1)} \le -\lambda$ and $|\hat{b}_1^{(1)}| \le M$ on Γ_1 and that $V \ge C_2$ in $\overline{\Omega}$. Also, $w' > 0$ on $(0, h)$ by (4.5.126). Then

$$\hat{\mathcal{B}}^{(1)}V = \hat{b}_1^{(1)}w' + \hat{b}_0^{(1)}V \le Mw' - \lambda C_2 \le -M \le g_1 \quad \text{on } \Gamma_1,$$

if C_2 is large. Then following the argument in the second half of p.169, we obtain $|u| \le V$ in Ω. This proves (4.5.119).

Next we prove (4.5.120). We continue to use $w(x_1)$ defined by (4.5.128) with $\hat{M} = 2\frac{M}{\lambda\varepsilon}$, but choose C_1 large so that $w(\frac{\varepsilon}{2}) \ge C$, where C is from (4.5.119). Let

$$v(x_1, x_2) = w(x_1).$$

We show that $|u| \le v$ in $\Omega_{\varepsilon/2} := \Omega \cap \{0 < x_1 < \varepsilon/2\}$. By the choice of C_1 above, we have $|u| \le v$ on $\partial\Omega_{\varepsilon/2} \cap \{x_1 = \varepsilon/2\}$. Also, $u = 0 = v$ on Γ_0. Finally, since all properties in (4.5.125) hold on $\Gamma_1 \cap \{0 < x_1 < \varepsilon/2\}$, the argument on pp. 168-169 shows that, after possibly increasing C_1, we have $\mathcal{L}(v) - f < 0$ in $\Omega_{\varepsilon/2}$, and $\mathcal{B}^{(k)}v \le g_k$ on $\Gamma_k \cap \partial\Omega_{\varepsilon/2}$ for $k = 1, 3$, where $g_3 = 0$. Then following the argument in the second half of p.169, we obtain $|u| \le v$ in $\Omega_{\varepsilon/2}$. From this, using that $v(0, x_2) = 0$ and $v_{x_1} \le \hat{M}C_1$ in $\Omega_{\varepsilon/2}$ by (4.5.128), we obtain (4.5.120) in $\Omega_{\varepsilon/2}$. Combining with (4.5.119), we obtain (4.5.120) in Ω.

New added Section 16.7.

16.7. Compactness of admissible solutions in the general case.

Fix $\theta_{\rm w}^* \in (\theta_{\rm w}^{\rm c}, \frac{\pi}{2})$ for the critical angle $\theta_{\rm w}^{\rm c}$ introduced in Definition 15.7.3.

Proposition 16.7.1 *All assertions proved in Section 11.6, which include Proposition 11.6.1 and Corollary 11.6.2, hold for both supersonic and subsonic admissible solutions, with only the following notational changes:* $\Gamma_{\rm sonic}$ *need to be replaced by* $\overline{\Gamma_{\rm sonic}}$ *in (11.6.2) and (11.6.3), where we recall that* $\overline{\Gamma_{\rm sonic}}$ *denotes* $\{P_0\}$ *for subsonic/sonic wedge angles.*

Proof. The proofs of Proposition 11.6.1 and Corollary 11.6.2 work with several minor changes in the case when $\theta_w^{(\infty)}$ is a subsonic or sonic angle, by using that all results on convergence, ellipticity and regularity which are used in their proof are extended to the present case.

For the proof of Proposition 11.6.1, we use that Corollaries 9.2.5, 9.6.6 and Proposition 10.6.1 were extended to the case of both supersonic and subsonic admissible solutions in Proposition 15.2.2, Corollary 15.6.3, and Proposition 15.7.5. We also use the results on regularity near sonic arc for supersonic and supersonic-near-sonic cases §16.3-16.4 and regularity near point P_0 for the subsonic-near-sonic and subsonic-away-from-sonic cases in §16.5-16.6. We describe the changes which require some additional the argument.

The argument in the paragraph starting from line -2 of page 434 still holds if $\theta_w^{(\infty)}$ is a supersonic wedge angle, with use of (16.4.59) in Proposition 16.4.6 instead of (11.4.4) in Proposition 11.4.6, and shows that $D\varphi^{(\infty)} = D\varphi_2^{(\infty)}$ on $\overline{\Gamma_{\rm sonic}^{(\infty)}}$. Furthermore, if $\theta_w^{(\infty)}$ is a subsonic or sonic angle, then, noting that $\psi^{(\infty)} \in C^{1,\alpha}(\overline{\Omega \cap B_\varepsilon(P_0)})$ by Propositions 16.5.3 and 16.6.11, and that $\varphi^{(\infty)}$ is a weak solution of Problem 2.6.1 by Corollary 9.2.5 (extended by Proposition 15.2.2), we obtain $D\varphi^{(\infty)}(P_0) = D\varphi_2^{(\infty)}(P_0)$.

In the argument of Step 2 of proof of Proposition 11.6.1, we use the fact that there exist $C > 0$ and $\alpha \in (0,1)$ such that any admissible solution φ for any wedge angle (supersonic and subsonic) $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2})$ satisfies

$$\|\varphi\|_{C^{1,\alpha}(\overline{\Omega})} \le C \qquad (*)$$

and its shock function $f_{O_1,{\rm sh}}$, introduced in Corollary 10.5.1 (extended in Proposition 15.7.1 to the case of all supersonic and subsonic wedge angles), satisfies

$$\|f_{O_1,{\rm sh}}\|_{C^{1,\alpha}([\theta_{P_1},\theta_{P_1}-])} \le \hat{C},$$

where $P_1 = P_0$ for subsonic (including sonic) wedge angles. Note that the last estimate extends (11.4.39) to the case of all supersonic and subsonic wedge angles. Both estimates follow by combining Corollaries 16.4.8, 16.5.4 and 16.6.12. With these estimates, argument of Step 2 of proof of Proposition 11.6.1, applies to any wedge angle $\theta_{\rm w}^{(\infty)} \in [\theta_{\rm w}^*, \frac{\pi}{2})$, where the argument for subsonic and sonic $\theta_{\rm w}^{(\infty)}$ does not include the case $\boldsymbol{\xi}_\infty \in \Gamma_{\rm sonic}$. Step 2 shows that (iii) of Proposition 11.6.1 holds for any $\boldsymbol{\xi}_\infty \in \Omega^{(\infty)} \setminus \{P_1^{(\infty)}, \dots, P_4^{(\infty)}\}$, where $P_1^{(\infty)} = P_4^{(\infty)} = P_0^{(\infty)}$ for subsonic and sonic wedge angles.

The argument of Step 3 of the proof of Proposition 11.6.1 is unchanged for the cases $\boldsymbol{\xi}_\infty = P_2^{(\infty)}$ and $\boldsymbol{\xi}_\infty = P_3^{(\infty)}$, since Proposition 10.5.1 is extended to the case of general (supersonic and subsonic) wedge angles in Proposition 15.7.5.

It remains to consider the cases when
- $\theta_{\rm w}^{(\infty)}$ is a subsonic (or sonic) wedge angle and $\boldsymbol{\xi}_\infty = P_0^{(\infty)}$;
- $\theta_{\rm w}^{(\infty)}$ is a supersonic wedge angle and $\boldsymbol{\xi}_\infty \in \{P_1^{(\infty)}, P_4^{(\infty)}\}$.

Note that if $\theta_{\rm w}^{(\infty)}$ is sonic, then $\theta_{\rm w}^{(i_j)}$ may be either subsonic or supersonic. It is sufficient to extend solutions $\varphi^{(\infty)}$, $\varphi^{(i_j)}$ to $B_R(\overline{\Gamma_{\rm sonic}^{(i_j)}})$, $B_R(\overline{\Gamma_{\rm sonic}^{(\infty)}})$ resp. for some small $R > 0$ (where $\overline{\Gamma_{\rm sonic}} = \{P_0\}$ for subsonic and sonic wedge angles), so that the extensions satisfy uniform $C^{1,\alpha}$ bound, after that we can argue as in the previous cases.

We discuss such extension for any admissible solution φ for $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$. We note that all results in §8.2 are extended to all wedge angles by Proposition 15.2.1, where $P_1 = P_4 = P_0 = \overline{\Gamma_{\rm sonic}}$ for subsonic and sonic angles, and we use the structure of Ω in coordinates (S, T) along $(\boldsymbol{\nu}_{\rm w}, \boldsymbol{\tau}_{\rm w})$, given by (8.2.5) in Corollary 8.2.14. By (8.2.24) with $\mathbf{e} = \boldsymbol{\nu}_{\rm w}$, and by Lemma 8.2.11, there exists $\lambda > 0$ such that $f'_{\boldsymbol{\nu}_{\rm w},{\rm sh}}(T_{P_1}) \le -\lambda$ for all $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$. Using uniform $C^{1,\alpha}$ regularity of shock functions for all $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$, obtained by by combining Corollaries 16.4.8, 16.5.4 and 16.6.12, we have that there exists $\delta > 0$ such that $f'_{\boldsymbol{\nu}_{\rm w},{\rm sh}} \le -\frac{\lambda}{2}$ on $[T_{P_1} - 2\delta, T_{P_1}]$ for all $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$. If $\theta_{\rm w}$ is supersonic wedge angle, we also need to consider the interval (T_{P_1}, T_{P_4}), on which the curve $\partial\Omega \cap \{S > 0\}$ is the sonic arc which is given in (8.2.24) by the graph of function $f_{\boldsymbol{\nu}_{\rm w},{\rm so}}$ defined in Remark 8.2.12. For that, we recall that $c_2^{(\theta_{\rm w})} \ge \frac{1}{C} > 0$ for all $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$ and also the continuous dependence of P_1 on θ_w. From Remark 7.5.5(i, ii) by an elementary geometric argument we obtain that $T_{P_1} > |(u_2, v_2)|$ and $f'_{\boldsymbol{\nu}_{\rm w},{\rm sh}}(T_{P_1}) > f'_{\boldsymbol{\nu}_{\rm w},{\rm so}}(T_{P_1})$. Then from the expression of $f_{\boldsymbol{\nu}_{\rm w},{\rm so}}$, it follows that $f'_{\boldsymbol{\nu}_{\rm w},{\rm sh}} \le -\frac{\lambda}{2}$ on (T_{P_1}, T_{P_4}). Thus we showed that for every wedge angle $\theta_{\rm w} \in [\theta_{\rm w}^*, \frac{\pi}{2}]$, it holds

$$\partial\Omega \cap \{S > 0,\ T > T_{P_1} - \delta\} = \{(S,T) \mid S = h(T),\ T \in (T_{P_1} - \delta, T_{P_4})\},$$

where $h \in C^0([T_{P_1} - \delta, T_{P_4}]) \cap C^1([T_{P_1} - \delta, T_{P_4}) \setminus \{T_{P_1}\})$ with $h' \le -\frac{\lambda}{2}$ for all $T \in [T_{P_1} - \delta, T_{P_4}) \setminus \{T_{P_1}\}$ and $h(T_{P_4}) = 0$.

Remark. For subsonic/sonic wedge angles, $P_1 = P_4 = P_0$ and $h = f_{\boldsymbol{\nu}_{\rm w},{\rm sh}}$ on $(T_{P_0} - \delta, T_{P_0})$. For supersonic wedge angles, $h = f_{\boldsymbol{\nu}_{\rm w},{\rm sh}}$ on $(T_{P_1} - \delta, T_{P_1})$ and $h = f_{\boldsymbol{\nu}_{\rm w},{\rm so}}$ on (T_{P_1}, T_{P_4}).

It follows that

$$\partial\Omega \cap \{S > 0,\ T > T_{P_1} - \delta\} = \{(S,T) \mid T = g(S),\ S \in (0,a)\}, \quad \text{where } a = f_{\boldsymbol{\nu}_{\rm w},{\rm sh}}(T_{P_1} - \delta),$$

and $\mathrm{Lip}[g] \le \frac{2}{\lambda}$ on $[0,a]$. Moreover, since $f'_{\boldsymbol{\nu}_{\rm w},{\rm sh}} \le -\frac{\lambda}{2}$ on $[T_{P_1} - 2\delta, T_{P_1}]$, then

$$\Omega^{(\delta)} := \{S \in (0,a),\ T_{P_1} - \delta < T < g(S),\ \} \subset \Omega.$$

Denote by $\hat\Omega$ the extension of Ω by reflection across $\{S = 0\}$, i.e. $\hat\Omega = (\overline{\Omega} \cup \hat\Omega^-)^0$, where $\hat\Omega^- = \{(S,T) \mid (-S,T) \in \Omega\}$. Let also, extend g to $(-a,a)$ by the even reflection $g(-S) = g(S)$ for $S \in [0,a)$. Then $\mathrm{Lip}[g] \le \frac{2}{\lambda}$ on $[-a,a]$, and

$$\hat\Omega^{(\delta)} := \{S \in (-a,a),\ g(S) > T > T_{P_1} - \delta\} \subset \hat\Omega.$$

Denote

$$\hat\Omega^{(\delta,\infty)} := \{S \in (-a,a),\ T > T_{P_1} - \delta\}.$$

Note that

$$\mathcal{N}_R(\overline{\Gamma_{\rm sonic}}) \subset \hat\Omega^{(\delta,\infty)}$$

for sufficiently small R depending only on λ and δ, and thus on the data and $\theta_{\rm w}^*$. Thus it remains to extend φ from $\hat\Omega^{(\delta)}$ to $\hat\Omega^{(\delta,\infty)}$ with the uniform control of $C^{1,\alpha}$ norm.

Since $\varphi_{\boldsymbol{\nu}_{\rm w}} = 0$ on $\Gamma_{\rm wedge}$, i.e. in $\varphi_S = 0$ on $\partial\Omega \cap \{S = 0\}$, we extend φ into $\hat\Omega$ by the even reflection $\varphi(-S,T) = \varphi(S,T)$ for $(S,T) \in \Omega$. Then

$$\|\varphi\|_{C^{1,\alpha}(\overline{\hat\Omega})} \le \|\varphi\|_{C^{1,\alpha}(\overline{\Omega})} \le C,$$

where the last inequality is from $(*)$, where C depends only on the data and $\theta_{\rm w}^*$. Now we extend φ from $\hat\Omega^{(\delta)}$ to $\hat\Omega^{(\delta,\infty)}$ using the extension operator introduced in Definition 13.9.3. Note that Lemma 13.9.6 shows $C^{2,\alpha}$ estimates for the extension operator, and the corresponding $C^{1,\alpha}$ estimates are obtained similarly (and easier). Thus, noting also the structure of domain $\hat\Omega^{(\delta)}$ and that $g(S) - (T_{P_1} - \delta) \ge \delta$, we obtain

$$\|\varphi\|_{C^{1,\alpha}(\overline{\hat\Omega^{(\delta,\infty)}})} \le C(\mathrm{Lip}[g], \alpha, \delta)\|\varphi\|_{C^{1,\alpha}(\overline{\hat\Omega^{(\delta)}})}.$$

Recalling that $\mathrm{Lip}[g], \alpha, \delta$ depend only on the data and θ_{w}^{*}, we obtain that, there exists $C > 0$ such that for R defined above and any admissible solution φ for $\theta_{\mathrm{w}} \in [\theta_{\mathrm{w}}^{*}, \frac{\pi}{2}]$, the extension defined above satisfies

$$\|\varphi\|_{C^{1,\alpha}(\overline{\mathcal{N}_R(\overline{\Gamma_{\mathrm{sonic}}})})} \leq C.$$

This completes the updates in the proof of Proposition 11.6.1.

For the proof of Corollary 11.6.2 for all types of wedge angles, we note that the results of §8.2 and §9.1 are extended to the case of both supersonic and subsonic admissible solutions in Propositions 15.1.1 and 15.2.2 respectively. We also change $\mathcal{N}_{\varepsilon}(\Gamma_{\mathrm{sonic}}^{(i)})$ and $\mathcal{N}_{\varepsilon}(\Gamma_{\mathrm{sonic}}^{(\infty)})$ to $\mathcal{N}_{\varepsilon}(\overline{\Gamma_{\mathrm{sonic}}^{(i)}}), \mathcal{N}_{\varepsilon}(\overline{\Gamma_{\mathrm{sonic}}^{(\infty)}})$, where we recall that $\overline{\Gamma_{\mathrm{sonic}}}$ denotes $\{P_0\}$ for subsonic/sonic wedge angles.