6.6 #4

Assume U is connected. A function $u \in H^1(U)$ is a weak solution of *Neumann's problem*

$$\begin{cases} -\Delta u = f & \text{in } U \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial U \end{cases} \tag{1}$$

if

$$\int_U Du \cdot Dv \, dx = \int_U fv \, dx$$

for all $v \in H^1(U)$. Suppose $f \in L^2(U)$. Prove (1) has a weak solution if and only if

$$\int_U f \, dx = 0.$$