2.5 #12

Suppose u is smooth and solves $u_t - \Delta u = 0$ in $\mathbb{R}^n \times (0, \infty)$.

(i) Show $u_\lambda(x,t) := u(\lambda x, \lambda^2 t)$ solves the heat equation for each $\lambda \in \mathbb{R}$.

(ii) Use (i) to show that $v(x,t) = x \cdot Du(x,t) + 2tu_t(x,t)$ solves the heat equation as well.