2.5 #5

We say $v \in C^2(\overline{U})$ is *subharmonic* if

$$-\Delta v \leq 0 \qquad \text{in } U.$$

(a) Prove for subharmonic v that

$$v(x) \leq \fint_{B(x,r)} v \, dy \qquad \text{for all } B(x,r) \subset U.$$

(b) Prove that $\max_{\overline{U}} v = \max_{\partial U} v$.

(c) Let $\phi : \mathbb{R} \to \mathbb{R}$ be smooth and convex. Assume u is harmonic, $v := \phi(u)$. Prove v is subharmonic.

(d) Prove $v := |Du|^2$ is subharmonic whenever u is harmonic.