HOMEWORK #8

1. Construct a map $f : X \to Y$ which induces trivial maps f_* in homology, but which is not nullhomotopic.

2. For finite CW complexes X and Y, show that

$$\chi(X \times Y) = \chi(X) \cdot \chi(Y).$$

3. If a finite CW complex X is a union of subcomplexes A and B, show that

$$\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B).$$

4. For a finite CW complex and $p : Y \to X$ an n-sheeted covering space, show that

$$\chi(Y) = n \cdot \chi(X).$$

5. Show that the closed orientable surface $T_g := T^2\# \cdots \#T^2$ (g times) of genus g is a covering space of T_h if and only if $g = n(h-1)+1$ for some non-negative integer n.

6. Show that if $f : \mathbb{RP}^{2n} \to Y$ is a covering map of a CW-complex Y, then f is a homeomorphism.

7. Calculate the homology of the 2-torus T^2 with coefficients in $\mathbb{Z}$, $\mathbb{Z}_2$ and $\mathbb{Z}_3$, respectively. Do the same calculations for the Klein bottle.

8. Is there a continuous map $f : \mathbb{RP}^{2k-1} \to \mathbb{RP}^{2k-1}$ with no fixed points? Explain.