

HOMEWORK #6**1.**

- (1) Show that the map $f : S^1 \rightarrow S^1$, $z \mapsto z^k$, has degree k .
- (2) Construct a map $f : S^n \rightarrow S^n$ of any given degree $k \in \mathbb{Z}$.

2. Construct a surjective map $S^n \rightarrow S^n$ of degree zero, for each $n \geq 1$.**3.** Let $f : S^n \rightarrow S^n$ be a map of degree zero. Show that there exist points $x, y \in S^n$ with $f(x) = x$ and $f(y) = -y$.**4.** Let $f : S^{2n} \rightarrow S^{2n}$ be a continuous map. Show that there is a point $x \in S^{2n}$ so that either $f(x) = x$ or $f(x) = -x$.**5.** Let X be the quotient space of S^2 under the identifications $x \sim -x$ for x in the equator S^1 . Compute the homology groups of X .