

HOMEWORK #4

1. Show that the map $p : S^1 \rightarrow S^1$, $p(z) = z^n$ is a covering. (Here we represent S^1 as the set of complex numbers z of absolute value 1.)
2. Let $p : E \rightarrow B$ be a covering map, with E path connected. Show that if B is simply-connected, then p is a homeomorphism.
3.
 - (1) Show that if $n > 1$ then any continuous map $f : S^n \rightarrow S^1$ is nullhomotopic.
 - (2) Show that any continuous map $f : \mathbb{R}P^2 \rightarrow S^1$ is nullhomotopic.
4.
 - (1) Classify all coverings of the Möbius strip up to equivalence.
 - (2) Show that every covering of the Möbius strip is homeomorphic to either \mathbb{R}^2 , $S^1 \times \mathbb{R}$ or the Möbius strip itself.
5.
 - (1) Show that the torus T^2 is a two-fold cover of the Klein bottle.
 - (2) Is it possible to realize the Klein bottle as a two-fold cover of itself?
 - (3) Find the universal cover of the Klein bottle.
6. Let $p : E \rightarrow B$ be a covering map with E simply-connected. Show that given any covering map $r : Y \rightarrow B$, there is a covering map $q : E \rightarrow Y$ such that $r \circ q = p$.
7. Let \mathbb{Z}_6 act on $S^3 = \{(z, w) \in \mathbb{C}^2, |z|^2 + |w|^2 = 1\}$ via $(z, w) \mapsto (\epsilon z, \epsilon w)$, where ϵ is a primitive sixth root of unity. Denote by L the quotient space S^3/\mathbb{Z}_6 .
 - (1) What is the fundamental group of L ?
 - (2) Describe all coverings of L .