HOMEWORK #3

1. Show that for n even S^n is not an H-*space*, i.e., there is no map $\mu : S^n \times S^n \to S^n$ so that $\mu \circ i_1 = id_{S^n}$ and $\mu \circ i_2 = id_{S^n}$, where i_1, i_2 are the inclusions on factors.

2. Using cup products, show that every map $S^{k+l} \to S^k \times S^l$ induces the trivial homomorphism $H_{k+l}(S^{k+l}) \to H_{k+l}(S^k \times S^l)$, assuming $k > 0$ and $l > 0$.

3. Describe $H^*(\mathbb{CP}^\infty/\mathbb{CP}^1; \mathbb{Z})$ as a ring with finitely many multiplicative generators. How does this ring compare with $H^*(S^6 \times \mathbb{HP}^\infty; \mathbb{Z})$?

4. Show that if $H_n(X; \mathbb{Z})$ is finitely generated and free for each n, then $H^*(X; \mathbb{Z}_p)$ and $H^*(X; \mathbb{Z}) \otimes \mathbb{Z}_p$ are isomorphic as rings, so in particular the ring structure with $\mathbb{Z}$-coefficients determines the ring structure with $\mathbb{Z}_p$-coefficients.

5. Show that the cross product map $H^*(X; \mathbb{Z}) \otimes H^*(Y; \mathbb{Z}) \to H^*(X \times Y; \mathbb{Z})$ is not an isomorphism if X and Y are infinite discrete sets. This shows the necessity of finite generation hypothesis in the Künneth theorem.