HOMEWORK #7

1. Compute $H_*(S^n \vee S^n \vee \cdots \vee S^n)$.

2. Let
$$Y := \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 x_2 = 0, x_2 \geq 0\}.$$
Is $Y \times \mathbb{R}$ homeomorphic to $\mathbb{R}^2$? Explain. (Hint: compute the local homology groups at the origin.)

3. Show that if $f : \mathbb{D}^n \to \mathbb{D}^n$ is a continuous map so that the restriction $f_{|S^{n-1}}$ is a homeomorphism $S^{n-1} \to S^{n-1}$, then f is surjective.

4. Show that the suspension ΣX of a space X has trivial cup product in positive degrees, i.e., $\alpha \cup \beta = 0$ for all $\alpha, \beta \in H^*(\Sigma X)$ of positive degrees.

5. Show that for n even S^n is not an H-*space*, i.e., there is no map $\mu : S^n \times S^n \to S^n$ so that $\mu \circ i_1 = id_{S^n}$ and $\mu \circ i_2 = id_{S^n}$, where i_1, i_2 are the inclusions on factors.

6. Show that if $f : \mathbb{RP}^{2n} \to Y$ is a covering map of a CW-complex Y, then f is a homeomorphism.

7. Are $S^2 \times S^2$ and $\mathbb{CP}^2 \# \mathbb{CP}^2$ homotopy equivalent? Explain.

8. Show that $H^*(\mathbb{RP}^5; \mathbb{Z}) \cong H^*(\mathbb{RP}^4 \vee S^5; \mathbb{Z})$ is a ring isomorphism, but $\mathbb{RP}^5$ is not homotopy equivalent to $\mathbb{RP}^4 \vee S^5$.