HOMEWORK #2

1. Show that the map $p : S^1 \to S^1$, $p(z) = z^n$ is a covering. (Here we represent S^1 as the set of complex numbers z of absolute value 1.)

2. Let $p : E \to B$ be a covering map, with E path connected. Show that if B is simply-connected, then p is a homeomorphism.

3. Let $p : E \to B$ be a covering, with E path connected. For $b \in B$, let $e \in p^{-1}(b)$. Show that the induced map $p_* : \pi_1(E, e) \to \pi_1(B, b)$ is injective.

4.

(1) Classify all coverings of the Möbius strip up to equivalence.
(2) Show that every covering of the Möbius strip is homeomorphic to either $\mathbb{R}^2$, $S^1 \times \mathbb{R}$ or the Möbius strip itself.

5. Let $\mathbb{Z}_6$ act on $S^3 = \{(z, w) \in \mathbb{C}^2,\ |z|^2 + |w|^2 = 1\}$ via $(z, w) \mapsto (\epsilon z, \epsilon w)$, where ϵ is a sixth root of unity.

(1) What is the fundamental group of the quotient $S^3/\mathbb{Z}_6$?
(2) Describe all coverings of the quotient $S^3/\mathbb{Z}_6$.