# STOCHASTIC CALCULUS AND THE MATHEMATICS OF FINANCE

LAURENTIU MAXIM

## 1. Syllabus

(1) Probability Theory.
   (a) Probability spaces and random variables.
   (b) Basic probability distributions.
   (c) Expectation and variance, moments.
   (d) Bivariate distributions.
   (e) Conditional probability.
(2) Derivatives.
   (a) What is a derivative security?
   (b) Types of derivatives.
   (c) The basic problem: How much should I pay for an option? Fair price.
   (d) Expectation pricing. Einstein and Bachelier, what they knew about derivative pricing? And what they didn't know?
   (e) Arbitrage and no arbitrage. The simple case of futures. Arbitrage arguments.
   (f) The arbitrage theorem.
   (g) Arbitrage pricing and hedging.
(3) Discrete time stochastic processes and pricing models.
   (a) Binomial models without much math. Arbitrage and reassigning probabilities.
   (b) A first look at martingales.
   (c) Stochastic processes, discrete in time.
   (d) Conditional expectations.
   (e) Random walks.
   (f) Change of probabilities.
   (g) Martingales.
   (h) Martingale representation theorem.
   (i) Pricing a derivative and hedging portfolios.
(4) Continuous time processes and their connection to PDE
   (a) Wiener processes.
   (b) Stochastic integration.
   (c) Stochastic differential equations and Ito's lemma.
   (d) Black-Scholes model.
   (e) Derivation of the Black-Scholes partial differential equation.
   (f) Solving the Black-Scholes equation. Comparison with martingale method.
   (g) Optimal portfolio selection.

## 2. References

(1) Steven Shreve: Stochastic Calculus for Finance I. The binomial asset pricing model
(2) Steven Shreve: Stochastic Calculus for Finance II. Continuous-time models