

NAME: _____

ID#: _____

PRACTICE FINAL EXAM

INSTRUCTIONS: *You have exactly two hours to complete the exam. You must show all your work in order to receive full credit. No calculators are allowed. You must obey the principles of academic integrity. You must include this sheet with your exam in order to receive a grade.*

1. Derive the formula

$$\int_0^a \left[\int_0^y e^{m(a-x)} f(x) dx \right] dy = \int_0^a (a-x) e^{m(a-x)} f(x) dx,$$

where a and m are constants, $a > 0$, and f is any continuous function on $[0, a]$.

2. Compute the area of the region cut from the plane $x + y + z = 2$ by the cylinder $x^2 + y^2 = 4$.

3. Evaluate the line integral

$$\int_C (y - z) dx + (z - x) dy + (x - y) dz,$$

where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + z = 1$.

4. If m is a positive constant, find that particular solution $y = f(x)$ of the differential equation

$$y''' - my'' + m^2y' - m^3y = 0$$

which satisfies the conditions: $f(0) = f'(0) = 0$, $f''(0) = 1$.

5.

(a) Find the general solution of the differential equation:

$$y^{(6)} + 8y^{(4)} + 16y'' = 0.$$

(b) Find the general solution of the equation

$$y'' + 4y = \sin(x)$$

on the interval $(-\infty, \infty)$.