COLLEGE GEOMETRY MATH 461

NAME: ______________________________

FINAL EXAM

INSTRUCTIONS: *The exam is due by email on May 16, 2013, no later than 5PM. Please email scanned copies of your exams to both Marci and myself.*
You may use your textbooks and notes, but may not work in groups on the exam.
You must show all your work in order to receive full credit.
You must obey the principles of academic integrity (using the Internet to find solutions for a certain problem is certainly not allowed!).

Solve FOUR of the following problems.

1. If $\triangle A_1B_1C_1$ and $\triangle A_2B_2C_2$ are situated so that the lines joining the corresponding vertices, A_1A_2, B_1B_2 and C_1C_2, are concurrent, show that the pairs of corresponding sides intersect in three collinear points. (Hint: Use Menelaus' Theorem.)

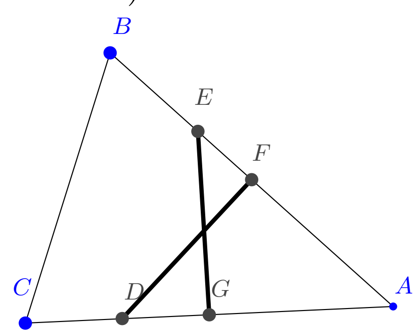
2. In $\triangle ABC$, points P, Q, and R are the midpoints of the sides AB, BC, and AC, respectively. Lines AN, BL, and CM are concurrent, meeting the opposite sides in N, L, and M, respectively. If PL meets BC at J, MQ meets AC at I, and RN meets AB at H, prove that H, I, and J are collinear.
(Hint: Use Ceva's and Menelaus' Theorem.)

3. Sides AB, BC, CD, and DA of a quadrilateral $ABCD$ are cut by a straight line at points K, L, M, and N, respectively. Prove that

$$\frac{BL}{LC} \cdot \frac{AK}{KB} \cdot \frac{DN}{NA} \cdot \frac{CM}{MD} = 1.$$

(Hint: Use Menelaus' Theorem.)

4. In a non-equilateral $\triangle ABC$, let D and E be the feet of the altitudes from B and C respectively, and let F and G denote the midpoints of AB and AC respectively. Show that the intersection point of EG and DF is on the Euler line (in other words it is on the line connecting O, H and G).
(Hint: Use Pappus' Theorem.)



5. From the point P on the circumcircle of $\triangle ABC$, perpendiculars PX, PY, and PZ are drawn to sides AC, AB, and BC, respectively. Prove that

$$PA \cdot PZ = PB \cdot PX.$$

(Hint: The use of the Simson line may come in handy.)