THE GLOBAL INVARIANT OF HYPERPLANE ARRANGEMENTS

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Abstract. A hyperplane in a complex vector space $\mathbb{C}^l$ is an $(l-1)$-dimensional affine subspace. A hyperplane arrangement $\mathcal{A}$ consists of finite hyperplanes in $\mathbb{C}^l$. The complement M of the union of the hyperplanes in $\mathcal{A}$ is an interesting topological space. The topological structure of M is one of the central topics in the theory of hyperplane arrangements. For example, the fundamental group $\pi(M)$ of M is an interesting and complicated invariant of M.

The lower central series of $\pi(M)$ is a chain of normal subgroups $G_1 = \pi(M), G_{k+1} = [G_k, G_1]$ for $k \geq 1$, where $[A, B]$ denotes the subgroup generated by commutators of elements in A and B. The ranks ϕ_k of the finitely generated abelian groups G_k/G_{k+1} are topological invariants of the arrangement $\mathcal{A}$. Falk called the third rank ϕ_3 *the global invariant* of $\mathcal{A}$, and posed the following problem. Give a combinatorial interpretation of ϕ_3. Falk also pointed out that this problem remains open, even for graphic matroids.

We solve Falk's problem for graphic hyperplane arrangements by proving the following formula. For graphic hyperplane arrangement $\mathcal{A}(G)$,

$$\phi_3 = 2(\#K_3 + \#K_4),$$

where $\#K_v$ is the number of cliques with v vertices in the graph G with which the arrangement $\mathcal{A}(G)$ associated.

By using the global invariant, we classified line arrangements in plane with at most six lines.

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