HOMEWORK 1

1. Let A be a real 3×3 matrix, with all entries positive. Show that A has a positive real eigenvalue. (Hint: Use Brower's fixed point theorem.)

2. Calculate the fundamental group of the spaces below:

(1) A 2-sphere with a diameter attached to it.
(2) A 2-sphere with the ecuatorial disc attached to it.
(3) The complement in $\mathbb{R}^3$ of a line and a circle. Note: There are two cases to consider, one where the line goes through the interior of the circle and the other where it doesn't. Are these two spaces homotopy equivalent?
(4) The complement in $\mathbb{R}^3$ of a line and a point not on the line.
(5) $\mathbb{R}^3 \setminus \{\mathrm{x-axis\ and\ y-axis}\}$.
(6) $\mathbb{R}^3$ minus two disjoint lines.
(7) $T^2 - \{x, y\}$, where x, y are two distinct points on the 2-torus.
(8) Moebius band. Are the cylinder and the Moebius band homeomorphic?

3. There are six ways to obtain a two-dimensional manifold by identifying pairs of sides in a square. In each case determine what surface one obtains.

4. What is the homotopy type of $\mathbb{R}P^2$ minus a point?

5. Let V be a finite dimensional vector space and W a subspace. Compute $\pi_1(V \setminus W)$.