NAME: ______________________________

FINAL EXAM

Due Friday 5/10 in class (or you can email it to me if you type your exam in Tex). You may use your texts and notes, but may not work in groups on the exam.

1. Use homotopy groups to show that there is no retraction $\mathbb{RP}^n \to \mathbb{RP}^k$ if $n > k > 0$.

2. Show that a closed simply-connected 3-manifold is homotopy equivalent to S^3. (Hint: Use Poincaré duality and the fact that closed manifolds are homotopy equivalent to CW complexes.)

3. Show that if $S^k \to S^m \to S^n$ is a fiber bundle, then $k = n - 1$ and $m = 2n - 1$.

4. Show that $\pi_7(S^4)$ is non-trivial.

5. Show that a map $f : X \to Y$ of connected CW complexes is a homotopy equivalence if it induces an isomorphism on π_1 and if a lift $\tilde{f} : \widetilde{X} \to \widetilde{Y}$ to the universal covers induces an isomorphism on homology.

6. Show that a CW complex is contractible if it is the union of an increasing sequence of subcomplexes $X_1 \subset X_2 \subset \cdots$ such that each inclusion $X_i \hookrightarrow X_{i+1}$ is nullhomotopic. Conclude that S^∞ is contractible, and more generally, this is true for the infinite suspension $\Sigma^\infty(X) := \bigcup_{n\geq 0} \Sigma^n(X)$ of any CW complex X.