INTRODUCTORY TOPOLOGY II MATH 752

NAME: ______________________________

FINAL EXAM

Due Friday 5/7 in class. You may use your texts and notes, but may not work in groups on the exam.

1. Let M be a closed, connected, oriented n-manifold and let $f : S^n \to M$ be a continuous map of non-zero degree, i.e., the morphism

$$f_* : H_n(S^n; \mathbb{Z}) \to H_n(M; \mathbb{Z})$$

is non-trivial. Show that M and S^n have the same $\mathbb{Q}$-homology.

2. Show that there is no orientation-reversing self-homotopy equivalence $\mathbb{CP}^{2n} \to \mathbb{CP}^{2n}$.

3. Suppose X is a CW complex with $\tilde{H}_i(X; \mathbb{Z}) = 0$ for all $i \geq 0$. Show that the suspension of X is contractible.

4. Show that a map $f : X \to Y$ of connected CW complexes is a homotopy equivalence if it induces an isomorphism on π_1 and if a lift $\tilde{f} : \tilde{X} \to \tilde{Y}$ to the universal covers induces an isomorphism on homology.

5. Show that $\pi_7(S^4)$ is non-trivial. [Hint: It contains a $\mathbb{Z}$-summand.]

6. Prove that the space $SO(3)$ of orthogonal 3×3 matrices with determinant 1 is homeomorphic to $\mathbb{RP}^3$.

7.

(a) Show that if $S^k \to S^m \to S^n$ is a fiber bundle, then $k = n-1$ and $m = 2n-1$.
(b) Show that if there were fiber bundles $S^{n-1} \to S^{2n-1} \to S^n$ for *all* n, then the groups $\pi_i(S^n)$ would be finitely generated free abelian groups computable by induction, and non-zero if $i \geq n \geq 2$.