HOMEWORK #4

1. Show that the map $p : S^1 \to S^1$, $p(z) = z^n$ is a covering. (Here we represent S^1 as the set of complex numbers z of absolute value 1.)

2. Let $p : E \to B$ be a covering map, with E path connected. Show that if B is simply-connected, then p is a homeomorphism.

3.

(1) Show that if $n > 1$ then any continuous map $f : S^n \to S^1$ is nullhomotopic.
(2) Show that any continuos map $f : \mathbb{RP}^2 \to S^1$ is nullhomotopic.

4.

(1) Classify all coverings of the Möbius strip up to equivalence.
(2) Show that every covering of the Möbius strip is homeomorphic to either $\mathbb{R}^2$, $S^1 \times \mathbb{R}$ or the Möbius strip itself.

5.

(1) Show that the torus T^2 is a two-fold cover of the Klein bottle.
(2) Is it possible to realize the Klein bottle as a two-fold cover of itself?
(3) Find the universal cover of the Klein bottle.

6. Let $p : E \to B$ be a covering map with E simply-connected. Show that given any covering map $r : Y \to B$, there is a covering map $q : E \to Y$ such that $r \circ q = p$.

7. Show that if G is a finite group with a fixed-point free action on a Hausdorff space X, the quotient map $p : X \to X/G$ is a covering.

8. Let $\mathbb{Z}_6$ act on $S^3 = \{(z, w) \in \mathbb{C}^2,\ |z|^2 + |w|^2 = 1\}$ via $(z, w) \mapsto (\epsilon z, \epsilon w)$, where ϵ is a primitive sixth root of unity. Denote by L the quotient space $S^3/\mathbb{Z}_6$.

(1) What is the fundamental group of L?
(2) Describe all coverings of L.
(3) Show that any continuous map $L \to S^1$ is nullhomotopic.