HOMEWORK #2

1. Calculate the fundamental group of the spaces below:

(1) $\mathbb{R}^3 \setminus \{x - \text{axis } \textit{and } y - \text{axis}\}$.
(2) The complement in $\mathbb{R}^3$ of a line and a point not on the line.
(3) $\mathbb{R}^3$ minus two disjoint lines.
(4) $T^2 \setminus \{x, y\}$, where x, y are two distinct points on the 2-torus.
(5) Möbius band. Are the cylinder and the Möbius band homeomorphic?
(6) The complement in $\mathbb{R}^3$ of a line and a circle. Note: There are two cases to consider, one where the line goes through the interior of the circle and the other where it doesn't. Are these two spaces homotopy equivalent?

2. Show that $\mathbb{RP}^3$ and $\mathbb{RP}^2 \vee S^3$ have the same fundamental group. Are they homeomorphic?

3. For a given a sequence of continuous maps

$$X_1 \xrightarrow{f_1} X_2 \xrightarrow{f_2} X_3 \xrightarrow{f_3} \cdots$$

define the quotient space

$$M := \left(\bigsqcup_{i \geq 1} X_i \times [0,1] \right) / \left((x_i, 1) \sim (f_i(x_i), 0) \right)$$

obtained from the disjoint union of cylinders $X_i \times [0,1]$ via the identification of $(x_i, 1) \in X_i \times \{1\}$ with $(f_i(x_i), 0) \in X_{i+1} \times \{0\}$. Compute the fundamental group of M in the case when each X_i is a circle S^1 and $f_i : S^1 \to S^1$ is the map $z \mapsto z^i$ (for each $i \geq 1$).