HOMEWORK #4

1. There are six ways to obtain a compact surface by identifying pairs of sides in a square. In each case determine what surface one obtains.

2. The following labeling schemes describe two dimensional surfaces:

- $abc^{-1}b^{-1}a^{-1}c$
- $abc^{-1}c^{-1}ba$
- $a_1a_2\cdots a_na_1{}^{-1}a_2{}^{-1}\cdots a_n{}^{-1}$

In each case determine what standard surface it is homeomorphic to.

3. Consider the space X obtained from a seven-sided polygonal region by means of the labeling scheme $abaaab^{-1}a^{-1}$. Show that $\pi_1(X)$ is the free product of two cyclic groups.

4. Let X be the quotient space obtained from an eight-sided polygonal region P by means of the labeling scheme $abcdad^{-1}cb^{-1}$. Let $\pi : P \to X$ be the quotient map.

- Show that P does not map all the vertices of P to the same point of X.
- Determine the space $A = \pi(\text{Bd } P)$ (the boundary of P), and calculate its fundamental group.
- Calculate the fundamental group of X. (Hint: first transform the labeling scheme into a standard one by cutting and pasting operations.)
- What surface is X hoemeomorphic to?

5. Let X be a space obtained by pasting the edges of a polygonal region together in pairs.

- Show that X is homeomorphic to exactly one of the spaces in the following list: S^2, $\mathbb{P}^2$, K, T_n, $T_n\#\mathbb{P}^2$, $T_n\#K$, where K is the Klein bottle and $n \geq 1$.
- Show that X is homeomorphic to exactly one of the spaces in the following list: S^2, $\mathbb{P}^2$, K_m, T_n, $\mathbb{P}^2\#K_m$, where K_m is the m-fold connected sum of K with itself and $m \geq 1$.