

HOMEWORK #3

1. There are six ways to obtain a compact surface by identifying pairs of sides in a square. In each case determine what surface one obtains.
2. The following labeling schemes describe two dimensional surfaces:
 - $abc^{-1}b^{-1}a^{-1}c$
 - $abc^{-1}c^{-1}ba$
 - $a_1a_2\cdots a_n a_1^{-1}a_2^{-1}\cdots a_n^{-1}$

In each case determine what standard surface it is homeomorphic to.

3. Consider the space X obtained from a seven-sided polygonal region by means of the labeling scheme $abaaab^{-1}a^{-1}$. Show that $\pi_1(X)$ is the free product of two cyclic groups.
4. Let X be the quotient space obtained from an eight-sided polygonal region P by means of the labeling scheme $abcdad^{-1}cb^{-1}$. Let $\pi : P \rightarrow X$ be the quotient map.
 - Show that P does not map all the vertices of P to the same point of X .
 - Determine the space $A = \pi(\text{Bd } P)$ (the boundary of P), and calculate its fundamental group.
 - Calculate the fundamental group of X . (Hint: first transform the labeling scheme into a standard one by cutting and pasting operations.)
 - What surface is X homeomorphic to?
5. Let X be a space obtained by pasting the edges of a polygonal region together in pairs.
 - Show that X is homeomorphic to exactly one of the spaces in the following list: S^2 , \mathbb{P}^2 , K , T_n , $T_n\#\mathbb{P}^2$, $T_n\#K$, where K is the Klein bottle and $n \geq 1$.
 - Show that X is homeomorphic to exactly one of the spaces in the following list: S^2 , \mathbb{P}^2 , K_m , T_n , $\mathbb{P}^2\#K_m$, where K_m is the m -fold connected sum of K with itself and $m \geq 1$.

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6. Show that $T^2 \# \mathbb{P}^2$ is homeomorphic to $K \# \mathbb{P}^2$ by using the algorithm of the classification theorem for compact surfaces.