HOMEWORK #2

1. Show that any two maps from an arbitrary space to a contractible space are homotopic. As a consequence, prove that if X is a contractible space, then any point in X is a deformation retract of X.

2. Let $A \subset X$ and suppose that $r : X \to A$ is a retraction of X onto A. If $a_0 \in A$, show that $r_* : \pi_1(X, a_0) \to \pi_1(A, a_0)$ is surjective.

3. Using the fact that the fundamental group of the circle S^1 is $\mathbb{Z}$, show that there are no retractions $r : X \to A$ in the following cases:

(1) $X = \mathbb{R}^3$, with A any subspace homeomorphic to S^1.
(2) $X = S^1 \times D^2$, with A its boundary torus $S^1 \times S^1$.
(3) X is the Möbius band and A its boundary circle.

4. Let A be a real 3×3 matrix, with all entries positive. Show that A has a positive real eigenvalue. (Hint: Use Brower's fixed point theorem.)

5. Show that if $f : \mathbb{D}^2 \to \mathbb{D}^2$ is a continuous map so that the restriction $f_{|S^1}$ is a homeomorphism $S^1 \to S^1$, then f is surjective.