HOMEWORK #1

1. Show that if $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then $k \circ h$ and $k' \circ h'$ are homotopic.

2. Let x_0 and x_1 be points of the path-connected space X. Show that $\pi_1(X, x_0)$ is abelian if and only if for every pair α and β of paths from x_0 to x_1, we have $\alpha_\# = \beta_\# : \pi_1(X, x_0) \to \pi_1(X, x_1)$. (Recall that $\alpha_\# : \pi_1(X, x_0) \to \pi_1(X, x_1)$ is the group isomorphism defined by $\alpha_\#([\gamma]) := [\alpha^{-1} * \gamma * \alpha]$.)

3. Let A be a subspace of $\mathbb{R}^n$; let $h : (A, a_0) \to (Y, y_0)$ be a continuous map of pointed spaces. Show that if h is extendable to a continuous map of $\mathbb{R}^n$ into Y, then h induces the trivial homomorphism on fundamental groups (i.e., h_* maps everything to the identity element).

4. Show that if X and Y are path-connected spaces, and $x \in X$, $y \in Y$, then $\pi_1(X \times Y, (x, y))$ is isomorphic to $\pi_1(X, x) \times \pi_1(Y, y)$.

5. Let V be a finite dimensional vector space and W a subspace.
Compute $\pi_1(V \setminus W)$.

6. What is the fundamental group of $\mathbb{RP}^2$ minus a point?