MATH 552 **FINAL EXAM** **SPRING** 2014

NAME: ______________________________

INSTRUCTIONS: *The exam is due via e-mail by 11:59 PM on Monday, May* 12*th. It is preferable that you type your exam in LATEX. Alternatively, you can scan your exam and e-mail it to me at* **maxim@math.wisc.edu** *with the subject line "Final Exam".*
You can also hand in your exam at any time during the week of May 5–9. Just slide your exam under my office door.
You must include this sheet with your exam in order to receive a grade.
You must show all your work in order to receive full credit.
You must obey the principles of academic integrity.

I. (50 points) Let A be the annulus in the plane consisting of the set

$$A := \{(x,y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 4\}.$$

Let S denote the surface obtained from A by identifying antipodal points of the inner circle and by identifying antipodal points of the outer circle. Compute $\pi_1(S)$ and write S as a connected sum of tori and projective planes.

II. (80 points) For a given sequence of continuous maps

$$X_1 \xrightarrow{f_1} X_2 \xrightarrow{f_2} X_3 \xrightarrow{f_3} \cdots$$

define the quotient space

$$M := \left(\bigsqcup_{i \geq 1} X_i \times [0,1] \right) / \big((x_i, 1) \sim (f_i(x_i), 0) \big)$$

obtained from the disjoint union of cylinders $X_i \times [0,1]$ via the identification of $(x_i, 1) \in X_i \times \{1\}$ with $(f_i(x_i), 0) \in X_{i+1} \times \{0\}$. Compute the fundamental group of M in the case when each X_i is a circle S^1 and $f_i : S^1 \to S^1$ is the map $z \mapsto z^i$ (for each $i \geq 1$).

III. (80 points) Let $\mathbb{Z}_6$ act on $S^3 = \{(z,w) \in \mathbb{C}^2,\ |z|^2 + |w|^2 = 1\}$ via $(z,w) \mapsto (\epsilon z, \epsilon w)$, where ϵ is a primitive sixth root of unity. Denote by L the quotient space $S^3/\mathbb{Z}_6$.

(1) What is the fundamental group of L?
(2) Describe all coverings of L.
(3) Show that any continuous map $L \to S^1$ is nullhomotopic.

IV. (40 points) Compute the homology of the Klein bottle.