

NAME: _____

ID#: _____

PRACTICE MIDTERM EXAM

INSTRUCTIONS: *You have exactly 50 minutes to complete the exam. You must show all your work in order to receive full credit. No calculators are allowed. You must obey the principles of academic integrity. You must include this sheet with your exam in order to receive a grade.*

1. Calculate the line integral $\int_C \frac{dx+dy}{|x|+|y|}$, where C is the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$, traversed once in a counter-clockwise direction.

2. A wire has the shape of the circle $x^2 + y^2 = a^2$. Determine its mass and moment of inertia about a diameter if the density at (x, y) is $|x| + |y|$.

3. Consider the following vector field:

$$f(x, y, z) = (x + z)\mathbf{i} - (y + z)\mathbf{j} + (x - y)\mathbf{k}.$$

Is f a gradient vector field? If so, find a corresponding potential function φ .

4. Let f be defined on the rectangle $Q = [0, 1] \times [0, 1]$ as follows:

$$f(x, y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases}$$

Prove that the double integral $\iint_Q f$ exists. What is its value?