NAME: ——————————————————

ID#: ——————————————————

PRACTICE FINAL EXAM

INSTRUCTIONS: *You have exactly two hours minutes to complete the exam. You must show all your work and be clear in your reasoning in order to receive full credit. No calculators, phones or tablets are allowed. You must obey the principles of academic integrity. You must include this sheet with your exam in order to receive a grade.*

1. Consider the linear space V of all 3×3 matrices.
  (a) What is the dimension of V?
  (b) Consider the set of all 3×3 symmetric matrices, which have the property that $a_{ij} = a_{ji}$ for all $i, j = 1, 2, 3$. Show that S is a subspace of V and compute its dimension.

2. Find a formula for the determinant of the matrix
$$A = \begin{pmatrix} B & O \\ C & D \end{pmatrix}$$
where B, C, D denote square matrices and O denotes of matrix of zeros.

3.
  (a) Prove that a square matrix and its transpose have the same eigenvalues.
  (b) Prove that a square matrix A is nonsingular if and only if 0 is not an eigenvalue of A.
  (c) If A is a nonsingular matrix, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

4.
  (a) Find the general solution of the differential equation:
$$y^{(6)} + 8y^{(4)} + 16y'' = 0.$$
  (b) Find the general solution of the equation
$$y'' + 4y = \sin(x)$$
  on the interval $(-\infty, \infty)$.