

Problem 1. (25 points). Solve the following ODEs. a) (10 points). $(x^4 + 2)\frac{dy}{dx} = yx^3$.

b) (15 points).
$$x\frac{dy}{dx} + 3y = 2x^5$$
, $y(1) = 1$.

Problem 2.(25 points). A 200 Liter tank initially contains 100 Liters of salt water with 20 kg of dissolved salt. A solution with 1 kg/L concentration of salt enters the tank at a rate of 3 L/min, mixes instantaneously with the solution in the tank, and exits the tank at a rate of 1 L/min.

a) (5 points). Write down the corresponding ODE initial value problem for the amount x(t) of salt in the tank at time t.

b) (20 points). How much salt is in the tank when the tank is full?

Problem 3.(25 points). Use elementary row operations (and clearly indicate every operation used) to solve the linear system:

$$2x_1 + 8x_2 + 3x_3 = 2$$

$$x_1 + 3x_2 + 2x_3 = 5$$

$$2x_1 + 7x_2 + 4x_3 = 8.$$

Problem 4.(25 points). Show that the 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is row equivalent to the 2 \times 2 identity matrix provided that $ad-bc\neq 0.$