

Mathematics 221, Lecture 2
Instructor: L. Maxim

Name: _____
TA's Name: _____

PRACTICE EXAM II

Do all six of the following problems. Show all your work, and write neatly.

No.	Points		Score
1	15		
2	20		
3	15		
4	20		
5	15		
6	15		
	100	TOTAL POINTS	

Problem I (15 points)

Show that the function $f(x) = x^3 + \frac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

Problem II (20 points)

Determine where the curve $y = \frac{(x+1)^2}{1+x^2}$ is increasing, decreasing, concave up and concave down. Where are its local extrema and inflection points? Use this information to sketch the curve.

Problem III (15 points) Evaluate the following limits:.

a) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$

b) $\lim_{x \rightarrow \frac{\pi}{2}} (x - \frac{\pi}{2}) \cot(x - \frac{\pi}{2})$

c) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$.

Problem IV (20 points)

Find the height and radius of the largest (in the sense of volume) right circular cylinder that can be put in a sphere of radius $\sqrt{3}$.

Problem V (15 points) Evaluate:

a) $\int_0^1 \frac{10\sqrt{x}}{(1+x^{3/2})^2} dx$

b) $\int_0^{\pi/2} \frac{\sin x}{(3+2 \cos x)^2} dx$

c) $\int_{-1}^1 x\sqrt{1-x^2} dx.$

Problem VI (15 points)

Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x -axis.