Mathematics 221, Lecture 1 **Name:** _____
Instructor: L. Maxim **TA's Name:** _____

PRACTICE FINAL EXAM

Do all eight of the following problems. Show all your work, and write neatly.

No.	Points		Score
1	25		
2	25		
3	25		
4	25		
5	25		
6	25		
7	25		
8	25		
	200	TOTAL POINTS	

Problem I (25 points)
Find the equation of the tangent line to the curve $x^2y^2+4xy = 12y$ at the point $(x, y) = (2, 1)$.

Answer: $y - 1 = -2(x - 2)$.

Problem II (25 points)
Use logarithmic differentiation to calculate the derivative $\frac{dy}{dx}$ of

$$y = \frac{2(x^2+1)}{\sqrt{\cos 2x}}.$$

Answer: $\frac{dy}{dx} = \frac{2(x^2+1)}{\sqrt{\cos(2x)}} \left(\frac{2x}{x^2+1} + \tan(2x) \right)$.

Problem III (25 points) Evaluate the following limits:.

a) $\lim_{x\to 1} \frac{\ln x^2}{x^2-1}$

Answer: 1.

b) $\lim_{x\to 0} \frac{2^{\sin x}-1}{e^x-1}$

Answer: $\ln 2$.

c) $\lim_{x\to 0^+} \left(1+\frac{3}{x}\right)^x$

Answer: 0.

Problem IV (25 points)
Find the volume of the solid obtained by revolving the curve $y = 2^x$ with $0 \leq x \leq 1$ about the x-axis.

Answer: $\frac{3\pi}{2 \ln 2}$.

Problem V (25 points)
Let R be the region in the first quadrant bounded on the left by the y-axis and on the right by the graphs of $y = \cos x$ and $y = \sin x$.

a) Compute the area of the region R.

Answer: $\frac{2}{\sqrt{2}} - 1$.

b) Set up an integral (but do not compute) for the volume of the solid obtained by rotating the region R about the y-axis.

Answer: $V = \int_0^{\pi/4} 2\pi x(\cos x - \sin x)dx$.

Problem VI (25 points) Evaluate the following integrals:

a) $\int_0^1 xe^{x^2}\ dx$

Answer: $\frac{e-1}{2}$.

b) $\int_1^e \frac{\sqrt{\ln x}}{x}\ dx$

Answer: $\frac{2}{3}$.

c) $\int_0^1 (x+3)e^{x^2+6x}\ dx$

Answer: $\frac{e^7-1}{2}$.

Problem VII (25 points)
The sum of two nonnegative numbers is 20. Find the numbers if one number plus the square root of the other is to be as large as possible.

Answer: The numbers are $\frac{1}{4}$ and $19\frac{3}{4}$.

Problem VIII (25 points)
Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

Answer: $\frac{2}{27}(10\sqrt{10} - 1)$.