Mathematics 221, Lecture 1 **Name:** _____
Instructor: L. Maxim **TA's Name:** _____

FINAL EXAM

Do all eight of the following problems. Show all your work, and write neatly. Justify your answers: *answers without supporting work will not receive full credit.*
It is not necessary to simplify your answers.

No.	Points		Score
1	25		
2	25		
3	25		
4	25		
5	25		
6	25		
7	25		
8	25		
	200	TOTAL POINTS	

Problem I (*25 points*) Show that the point $(2, 4)$ lies on the curve $x^3 + y^3 = 9xy$. Then find the tangent and normal to the curve at this point.

Problem II (*25 points*) Calculate the derivative $\frac{dy}{dx}$ of $y = x^{\sin x}$.

Problem III (*25 points*) Evaluate the following limits:

a) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

b) $\lim_{x\to\infty} x^{1/x}$

c) $\lim_{x\to\infty} \left(\frac{x+2}{x-1}\right)^x$

Problem IV (*25 points*) The region between the curve $y = 1/x^2$ and the x-axis from $x = 1/2$ to $x = 2$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

Problem V (*25 points*) Find the area of the region enclosed by the curves $y = 2\sin x$ and $y = \sin 2x$, for $0 \leq x \leq \pi$.

Problem VI (*25 points*) Evaluate the following integrals:

a) $\int \frac{e^{1/x}}{x^2}\, dx$

b) $\int_2^4 \frac{1}{x(\ln x)^2}\, dx$

c) $\int \frac{\cos(1-\ln x)}{x}\, dx$

Problem VII (*25 points*) Find the rectangle with *largest* area inscribed into the circle of radius 1.

Problem VIII (*25 points*) Find the length of the curve

$$y = \int_{-2}^{x} \sqrt{9t^4 - 1}\, dt$$

from $x = -2$ to $x = -1$.