Mathematics 221, Lecture 2 **Name:** _____
Instructor: L. Maxim **TA's Name:** _____

PRACTICE EXAM I

Do all six of the following problems. Show all your work, and write neatly.

No.	Points		Score
1	15		
2	20		
3	15		
4	20		
5	15		
6	15		
	100	TOTAL POINTS	

Problem I (15 points) Show that the equation

$$x^3 - 15x + 1 = 0$$

has a solution in the interval $[-1, 1]$.

Problem II (20 points) Starting at time $t = 0$, a particle moves along the x-axis in such a way that at time t its position is given by $x = 27t - t^3$, where t is measured in seconds.

a) Find the average velocity of the particle during the time interval $0 \leq t \leq 2$.

b) The particle moves to the right for a while, reaches some furthest right point, and then starts turning to the left. What is the velocity and acceleration of the particle at the moments when it is at its furthest right point? When and where does that occur?

c) When does the particle reach the furthest right point?

Problem III (15 points) Evaluate the following limits:.

a) $\lim_{x\to 2} \frac{x^2-4}{\sqrt{x}-\sqrt{2}}$

b) $\lim_{x\to 0} \frac{\sin^2(x)}{x\tan(2x)}$

c) $\lim_{x\to 0} x^3 \sin^2(\frac{1}{x})$. (Hint: use the sandwich theorem.)

Problem IV (20 points) Find the asymptotes of the graph of

$$f(x) = \frac{x^2 - 3}{2x - 4},$$

then sketch its graph.

Problem V (15 points) Let

$$f(x) = \begin{cases} 1\ , & x \geq 0 \\ 1 - x\ , & x < 0. \end{cases}$$

a) Show f is continuous at every point in its domain.

b) Show $f'(0)$ does not exist.

Problem VI (15 points) Find equations for the tangents to the curve $y = x^3 - 4x + 1$ at the points of the slope of the curve is 8.