

Mathematics 211, Lecture 2
Instructor: L. Maxim

Name: _____
TA's Name: _____

PRACTICE FINAL EXAM

Do all five of the following problems. Show all your work, and write neatly.
Answers without full justification will only receive partial credit.
Use of books, notes, phones or calculators during the exam is NOT allowed.

No.	Points		Score
1	25		
2	25		
3	25		
4	25		
5	25		
6	25		
7	25		
8	25		
	200	TOTAL POINTS	

Problem I. (25 points)

Sketch the region enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ and find its area.

Problem II. (25 points)

a) Evaluate the integrals:

$$(i) \int_1^2 \frac{\ln x}{x^2} \quad (ii) \int_0^1 \frac{x^3}{\sqrt{4+x^2}}.$$

b) Suppose that $f(0) = 1$, $f(2) = 2$, $f'(0) = -1$, $f'(2) = 3$ and f'' is continuous. Find the value of $\int_0^2 x f''(x) dx$.

Problem III. (25 points)

A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

- (a) At what rate is his distance from second base decreasing when he is halfway to first base?
- (b) At what rate is his distance from third base increasing at the same moment?

Problem IV. (25 points)

Find the equation of the tangent line to the curve $\ln(y + x) = xy$ at the point $(1, 0)$.

Problem V. (25 points)

Sketch the graph of the function

$$f(x) = \frac{1}{1 + e^{-x}}$$

by first considering the domain, intervals of increase and decrease, local extreme points, concavity and inflection points, and end behaviour, including any vertical or horizontal asymptotes.

Problem VI. (25 points)

A window has the shape of a semicircle on the top of a rectangle. If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

Problem VII. (25 points)

Find an equation of the curve that passes through the point $(3, 10)$ and whose slope at (x, y) is x^2y^2 .

Problem VIII. (25 points)

Find the values of p for which the integral $\int_1^\infty \frac{1}{x^p} dx$ converges, and evaluate the integral for those values of p .