

ON VAGUE SETS AND VAGUE LOGIC

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Abstract. The subject matter of the consideration touches the problem of vagueness from the logical, set-theoretical and the computer science perspective. The paper proposes a new, formal-logical approach to the problem.

The starting point is the concept of agent's unit information about any object o of discovered reality with respect to a relation R defined on U . It is the image $\overline{R}(o)$ of the object o wrt R . If $\overline{R}(o)$ is unknown to the agent, then the unit information for him can be given as the equation:

$$(e) \quad \overline{R}(o) = X,$$

where X is an unknown quantity. Its scope is the at least a two-element family \mathbf{V}_o of sets (relations) that are possible solutions of (e) from the point of view of the agent. Then we say that the agent's unit information about o wrt R is *vague* and the family \mathbf{V}_o is a *vague set*.

A language representation of (e) is the vague sentence:

$$(re) \quad a \text{ is } V \text{ or } V(a),$$

where a is the singular term of o and V is the name-predicate (resp. vague-predicate) corresponding to X ; the denotation of V is the vague set \mathbf{V}_o - the family of all denotations (extensions) of sharp terms representing V from the agent's point of view. The upper and lower approximations (limits) of \mathbf{V}_o are algebraic boundaries in $P(U)$. Some operations on vague sets and their algebraic properties are also presented. Some important conditions about the membership relation for vague sets, in connection to Zadeh's fuzzy sets (1965), Pawlak's rough sets (1982) and Blizard's multisets (1989) are established as well. A view on the problem of logic of vague sentences (*vague logic*) based on vague sets is also discussed. The considerations intend to take into account a 'conservative', classical approach to reasoning based on vague premises.

AUTONOMOUS SECTION OF APPLIED LOGIC

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