**On Ershov Semilattices of Degrees of $\Sigma$-definability of Structures**

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The notion of $\Sigma$-definability of a structure in an admissible set, introduced by Yu.L. Ershov [1], is an effectivization of the model-theoretical notion of interpretability of structures, and, at the same time, a generalization of the notion of constructivizability of a structure on natural numbers. For structures $\mathfrak{M}$ and $\mathfrak{N}$, let $\mathfrak{M} \leqslant_\Sigma \mathfrak{N}$ means that $\mathfrak{M}$ is $\Sigma$-definable in $\mathbb{HF}(\mathfrak{N})$, the least admissible set over $\mathfrak{N}$. Preordering $\leqslant_\Sigma$, considered for structures of cardinality $\leqslant \alpha$, generates the upper semilattice $\mathcal{S}_\Sigma(\alpha)$. $\Sigma$-degrees of some uncountable structures (fields of real, p-adic and complex numbers, dense linear orders, etc.) were studied in [1,2,3].

We show that the semilattices of Turing and enumeration degrees are embeddable in a natural way into the semilattices of $\Sigma$-degrees, by means of $\Sigma$-degrees of structures having a degree (resp., $e$-degree). The notion of a structure having a degree, known in computable model theory, gives only a partial measure of complexity, since there are a lot of structures which do not have a degree. $\Sigma$-degrees, as well as degrees of presentability with respect to different effective reducibilities [4], are natural measures of complexity which are total, i.e. defined for any structure.

In this talk we consider some recent results on some local and global properties of Ershov semilattices [5].

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