Our Integration Toolbox So Far

Power Rule

$$\boxed{\frac{d}{dx}x^n = nx^{n-1}}$$

Reverse Power Rule

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

⚠ $n \neq -1$

Slogan: Shoes & Socks

Chain Rule

$$\boxed{(f\circ g)'(x) = f'(g(x))g'(x)}$$

U-Substitution (Reverse Chain Rule)

$$\boxed{\int f'(g(x))g'(x)\,dx = (f\circ g)(x)}$$

Slogan: the derivative of u must appear multiplied by dx

Product Rule

$$\boxed{(fg)'(x) = f(x)g'(x) + f'(x)g(x)}$$

Integration By Parts

(Reverse product rule)

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Slogan: - u's derivative is simpler than u
- the integral of dv does not get more complicated

Some clever ways to use this method

- Invisible dv
- Use it multiple times in a row
- use it multiple times to express your original integral in terms of itself e.g. $\int e^x \cos(x)\,dx$

"swallows itself"

Q: What about "Reverse Quotient Rule?"

A: Well, the quotient rule for derivatives

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

is actually just the product rule in disguise

Product Rule

$$\underbrace{\frac{d}{dx}\frac{f(x)}{g(x)}}_{\text{quotient}} = \frac{d}{dx}\underbrace{f(x)\cdot\frac{1}{g(x)}}_{\text{product}} = f(x)\underbrace{\frac{d}{dx}\frac{1}{g(x)}}_{\text{need to compute this derivative}} + \left(\frac{d}{dx}f(x)\right)\frac{1}{g(x)}$$

if we set $h(x) = \frac{1}{x}$ then $\frac{1}{g(x)} = h(g(x)) = (h \circ g)(x)$

chain rule

$$\text{So } \frac{d}{dx}\frac{1}{g(x)} = \underbrace{\frac{d}{dx}(h \circ g)(x)}_{\text{derivative of a composition of functions}} = h'(g(x))g'(x)$$

Just need to find $h'(x)$ now

$$h'(x) = \frac{d}{dx}h(x) = \frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = -x^{-2} = \frac{-1}{x^2}$$

power rule w/ $n=-1$

Putting everything together now

Product Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = f(x)\frac{d}{dx}\frac{1}{g(x)} + f'(x)\frac{1}{g(x)}$$

Chain Rule

$$= -f(x)\frac{g'(x)}{[g(x)]^2} + f'(x)\frac{1}{g(x)}$$

find common denominators

Quotient Rule

$$= \frac{f'(x)\,g(x)}{g(x)\underbrace{g(x)}_{\text{multiply by 1 in a special way}}} - \frac{f(x)g'(x)}{[g(x)]^2} = \boxed{\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}}$$

So we do not need a separate rule for Reversing derivatives that require the quotient rule. Integration By Parts + u-substitution should be enough

Today: <u>Trigonometric Integrals</u>

$$\int \sin^m(x)\cos^n(x)\,dx$$ where m & n can be any exponents

Tomorrow:

$$\int \tan^m(x)\sec^n(x)\,dx$$ where again m & n can be any exponents

Products of Powers of Sine & Cosine

$$\int \sin^m(x)\cos^n(x)\,dx$$

Some easy examples:

calc I $\begin{cases} m=1 \\ n=0 \end{cases}$ $\qquad \int \sin(x)\,dx = -\cos(x) + C$

$\begin{cases} m=0 \\ n=1 \end{cases}$ $\qquad \int \cos(x)\,dx = \sin(x) + C$

either choice of u-sub $\begin{cases} m=1 \\ n=1 \end{cases}$ $\qquad \int \sin(x)\cos(x)\,dx$

take $u=\sin(x)$ or $u=\cos(x)$

$du=\cos(x)\,dx \qquad du=-\sin(x)\,dx$

$$\int u\,du = \frac{u^2}{2}+C \qquad -\int u\,du = -\frac{u^2}{2}+C$$

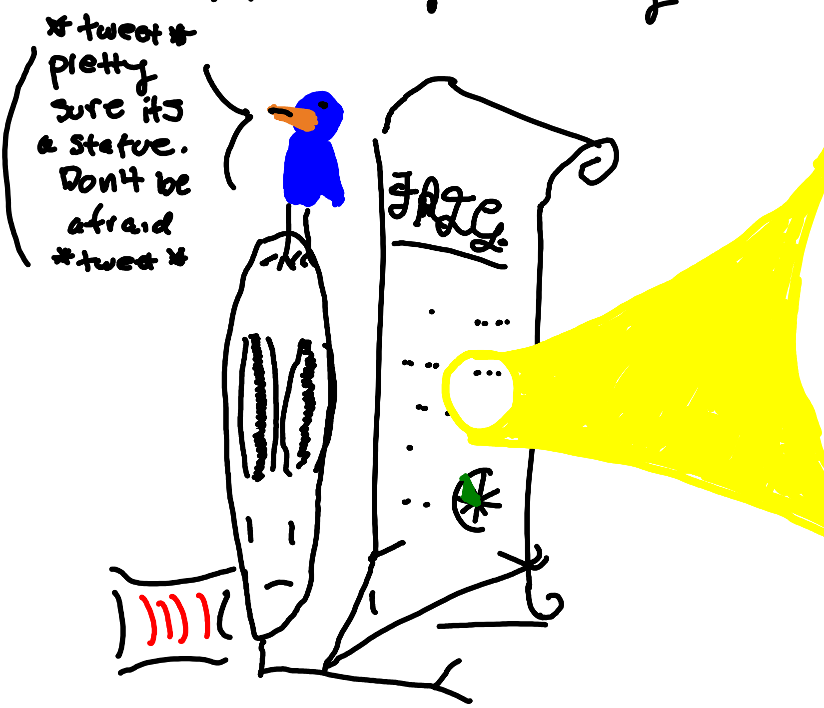
$$\boxed{= \frac{\sin^2(x)}{2}+C \qquad = -\frac{\cos^2(x)}{2}+C}$$

it might not be obvious that answers obtained from different solutions are in fact equal. The best way to check your work is by taking the derivative

The First Place We Get Stuck Happens When

$m=2$, $n=0$: $\int \sin^2(x)\,dx$ OR $m=0$, $n=2$: $\int \cos^2(x)\,dx$

We need a new idea. In this case, the idea comes from staring endlessly at a list of trig identities



Pythagorean Theorem

$$\sin^2\theta + \cos^2\theta = 1$$

Double Angle Formula (2 *)

$$\cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$$

From these we can deduce

$$1-\sin^2(\theta) = \cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$$

$$\Downarrow$$

$$\sin^2(\theta) = 1 - \frac{\cos(2\theta)+1}{2}$$

$$\boxed{\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}}$$

&

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

divide by $\cos^2(\theta)$

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\frac{a^c}{b^c} = \left(\frac{a}{b}\right)^c$$

algebra fact

$$\left(\frac{\sin(\theta)}{\cos(\theta)}\right)^2 + 1 = \left(\frac{1}{\cos(\theta)}\right)^2$$

definitions of trig functions

$$[\tan(\theta)]^2 + 1 = [\sec(\theta)]^2$$

these mean the same thing but the second way of writing it is prefered because it is more compact

$$\boxed{\tan^2(\theta) + 1 = \sec^2(\theta)}$$

The Main 4 Trig Identities

(for success in this course)

(PYT)

Pythagorean Theorem 1

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

divide by $\cos^2\theta$

Pythagorean Theorem 2

$$\boxed{\tan^2(\theta) + 1 = \sec^2\theta}$$

(PYT2)

Cosine double angle

C2∡

$$\boxed{\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}}$$

Sine double angle

S2∡

$$\boxed{\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}}$$

So far, you only need to memorize one thing

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

but we will learn how to come up with this identity in a way that's easy to remember (later) so it doesn't matter if you actually memorize it or not (in terms of the final).

When faced with $\int \sin^m x \cos^n x$ reach for your

PYT

to exchange even powers of sin for even powers of cos or the other way around

$$\sin^2(\theta) = 1-\cos^2(\theta)$$

$$\cos^2(\theta) = 1-\sin^2(\theta)$$

$$\sin^{2K}(\theta) = (1-\cos^2(\theta))^K$$

$$\cos^{2K}(\theta) = (1-\cos^2(\theta))^K$$

C2X

to exchange decrease exponent on cosine by 1 (provided it starts as 2 or larger)

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

$$\cos^n(\theta) = \cos^{n-2}(\theta)\left[\frac{1+\cos(2\theta)}{2}\right]$$

S2X

//

replacing cos w/ sin

//

$m=0$
$n=2$

$$\int \cos^2 x\, dx \underset{\text{C2*}}{=} \int \frac{1+\cos(2x)}{2}\, dx = \int \frac{1}{2}\, dx + \int \frac{\cos(2x)}{2}\, dx$$

$$= \frac{1}{2}x + \frac{\sin(2x)}{4} + C$$

Double check

$$\frac{d}{dx}\left(\frac{1}{2}x + \frac{\sin(2x)}{4} + C\right) = \frac{1}{2}\frac{dx}{dx} + \frac{1}{4}\frac{d}{dx}\sin(x) + \frac{d}{dx}C$$

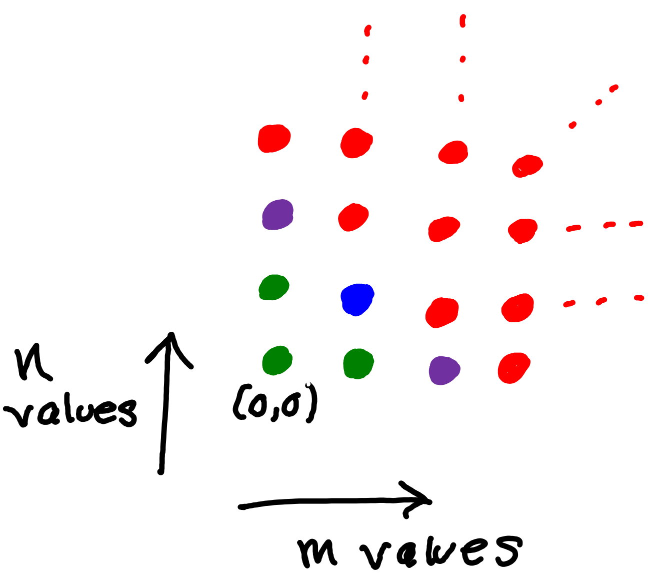
$$= \frac{1}{2} + \frac{1}{4}\cdot 2\cdot\cos(2x) + 0 = \frac{1}{2} + \frac{\cos(2x)}{2} \underset{\text{C2*}}{=} \cos^2 x \ \checkmark$$

Your Turn

$m=2$
$n=0$

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{\sin(2x)}{4} + C$$

What do we know so far? $\int \sin^m x \cos^n x\, dx$



■ = these integrals are easy or memorized (Calc I)

■ = u-Sub with either $u=\cos(\theta)$ or $u=\sin(\theta)$

■ = use the appropriate double angle formula

■ = have not figured out how to integrate these yet

(dots in this picture (& ones like it) represent integrals we either do or don't know how to solve. The color indicates its status.)

We will now start populating this grid with strategies much faster.

$m \neq 0$

$n = 1$

$$\int \sin^m(x) \underbrace{\cos(x)\,dx}_{\text{looks like a good candidate for } du}$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int u^m\,du$$

$$= \frac{u^{m+1}}{m+1} + C = \boxed{\frac{\sin^{m+1}(x)}{m+1} + C}$$

Convert this to the case above

$m \neq 0$

& Odd # cosines

$n = 2k+1$

$$\int \sin^m(x)\cos^{2k+1}(x)\,dx$$

$$= \int \sin^m(x)\underbrace{\cos^{2k}(x)}_{\text{use PYT to convert into sines}}\underbrace{\cos(x)\,dx}_{du}$$

PYT

$$= \int \sin^m(x)\underbrace{(1-\sin^2(x))^k}_{\text{expand this out \& we get a bunch of terms only involving powers of sine}}\cos(x)\,dx$$

$u = \sin x$

$$\left(= \int \underbrace{u^m(1-u^2)^k}_{\text{Polynomial}}\,du\right)$$

Each integral above is now of the previous type (some # of sines w/ only a single cosine)

Here is an explicit example to help you get the feel for this last step

$$\int \sin^4(x)\cos^5(x)\,dx$$

$m=4\neq 0$
$n=5=2\cdot 2+1$
(so $k=2$)

$$=\int \sin^4(x)\underbrace{\cos^4(x)}_{\text{"exchange" for sines}}\underbrace{\cos(x)dx}_{\text{"isolate" the odd cosine out"}}$$

$$=\int \sin^4(x)\left[\cos^2(x)\right]^2\cos(x)\,dx$$

$$\overset{\text{PYT}}{=}\int \sin^4(x)\left[1-\sin^2(x)\right]^2\cos(x)\,dx$$

$$=\int \sin^4(x)\left(1-\sin^2(x)\right)\left(1-\sin^2(x)\right)\cos(x)\,dx$$

$$=\int \sin^4(x)\left[1-2\sin^2(x)+\sin^4(x)\right]\cos(x)\,dx$$

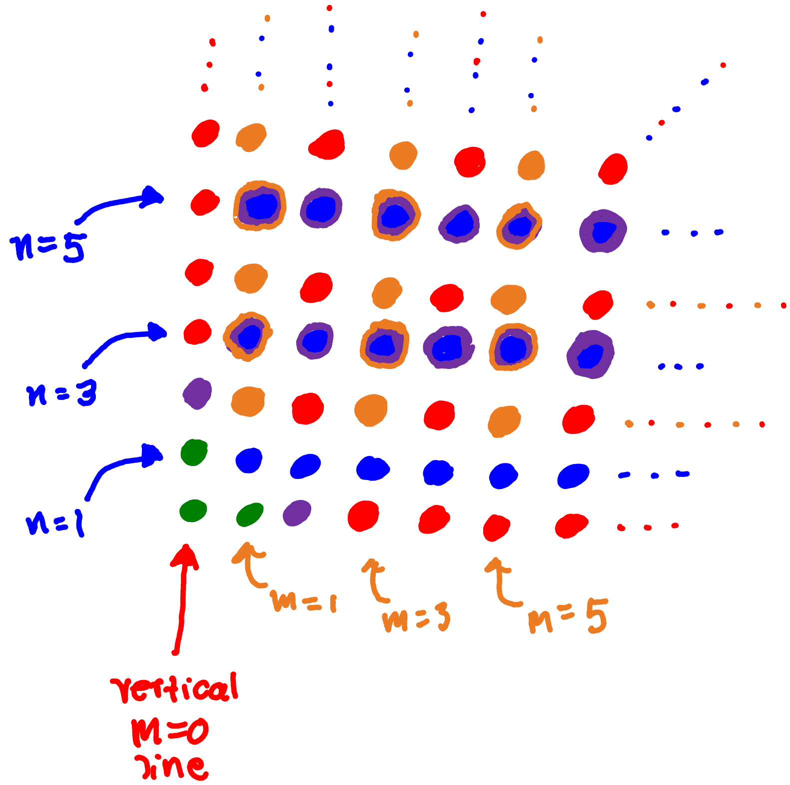
$$=\int \sin^4(x)\cos(x)\,dx-2\int \sin^6(x)\cos(x)\,dx+\int \sin^8(x)\cos(x)\,dx$$

set $u=\sin(x)$ for all 3 integrals

$du=\cos(x)dx$

$$=\int u^4du-2\int u^6du+\int u^8du=\frac{u^5}{5}-2\frac{u^7}{7}+\frac{u^9}{9}+C$$

$$\boxed{=\frac{\sin^5(x)}{5}-2\frac{\sin^7(x)}{7}+\frac{\sin^9(x)}{9}+C}$$



= easy or memorized

= double angle formulas

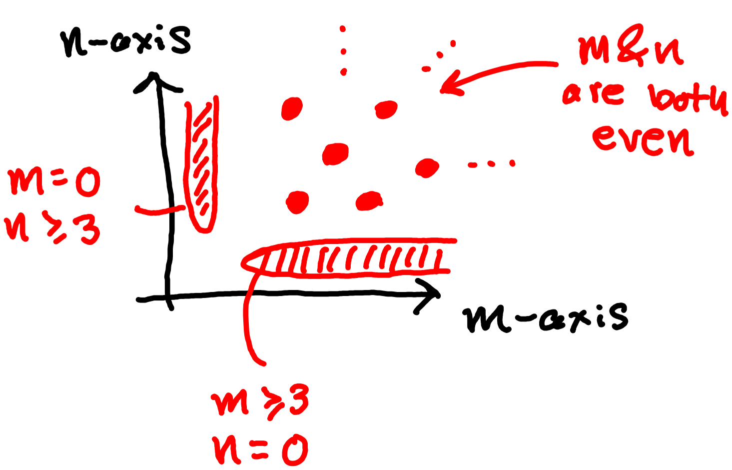
= let $u = \sin(x)$

= exchange all even powers of cos for even powers of sin then set $u = \sin(x)$

= you should be able to figure out how to integrate these now that we have seen similar examples

= the strategy for & will both work

What Remains?



$$\int \sin^{2M}(x)\cos^{2N}(x)\,dx$$

$$\int \sin^{m}(x)\,dx, \quad m \geq 3$$

$$\int \cos^{n}(x)\,dx, \quad n \geq 3$$

$\int \cos^n(x)\,dx$ Assuming $n \geq 3$ we can factor out a copy of cosine & still have at least 2 leftover

$$\| $$

$$\int \cos^{n-1}(x)\cos(x)\,dx \qquad n \geq 3 \Rightarrow n-1 \geq 2$$

Factoring out as many copies of $\cos^2(x)$ from $\cos^{n-1}(x)$ as possible ($n-1 \geq 2$ means there is always at least 1)

we are either left with

$$\int \cos^{n-1}(x)\cos(x)\,dx$$

even $n-1=2k$ OR even $n=2k$

$$\int [\cos^2(x)]^k \cos(x)\,dx \qquad \int [\cos^2(x)]^k\,dx$$

PYT $\|$ $\quad\quad$ $\|$ C2X

$$\underbrace{\int [1-\sin^2(x)]^k \cos(x)\,dx}_{\text{terms have lots of different values of } n} \qquad \underbrace{\int \left[\frac{1+\cos(2x)}{2}\right]^k dx}_{\text{every term (once expanded) is lower degree in terms of powers of cosine so the problem has gotten easier}}$$

terms have lots of different values of n

we do know how to solve these $\left\{ \begin{array}{ll} n=0 & m=1 \\ n\neq 0 & m=1 \end{array} \right.$

this is reduction formula territory

every term (once expanded) is lower degree in terms of powers of cosine so the problem has gotten easier

Now you should be able to reason similarly for powers of sin ≥ 3

Finally $\int \sin^{2M}(x)\cos^{2N}(x)\,dx$ when both are even powers

use $2\measuredangle$ laws to convert all sines to cosines with doubled input & all cosines to other cosines w/ doubled input

e.g.

$$\int \sin^{2M}x\cos^{2N}x\,dx$$

S2$\measuredangle$
Sine double angle formula

$$= \int \left(\frac{1-\cos(2x)}{2}\right)^M \cos^{2N}x\,dx$$

C2$\measuredangle$
cosine double angle formula

$$= \int \underbrace{\left(\frac{1-\cos(2x)}{2}\right)^M \left(\frac{1+\cos(2x)}{2}\right)^N}_{\text{simplifies to a polynomial in } \cos(2x) \text{ so this is now reduced to the case } \boxed{m = \text{anything},\ n = 0}} dx$$

Cosine Reduction Formula

$$\int \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

Integration by parts

(0,4)

$$\int \cos^4(x)\,dx = \int \underbrace{\cos^3(x)}_{u}\,\underbrace{\cos(x)\,dx}_{dv} = uv - \int v\,du$$

$du = -3\cos^2(x)\sin(x)\,dx$

$v = \sin(x)$

reduction

$$-\int v\,du = 3\int \cos^2(x)\sin^2(x)\,dx \overset{PYT}{=} 3\int \cos^2 x\,[1-\cos^2(x)]\,dx$$

$$\int \cos^4(x)\,dx = \sin(x)\cos^3(x) + 3\int \cos^2 x\,dx - 3\int \cos^4 x\,dx$$

$$\Longrightarrow \quad 4\int \cos^4(x)\,dx = \sin(x)\cos^3(x) + 3\int \cos^2 x\,dx$$

Integrate by parts

(0,3)

$$\int \cos^3(x)\,dx = \int \underbrace{\cos^2(x)}_{u}\,\underbrace{\cos(x)\,dx}_{dv} = uv \underbrace{-\int v\,du}_{+\,2\int \cos(x)\sin^2(x)\,dx}$$

$du = -2\cos(x)\sin(x)\,dx$

$v = \sin(x)$

$\boxed{\begin{matrix} n \neq 0 \\ m = 1 \end{matrix}}$

C&X

(0,2)

$$\int \cos^2(x)\,dx = \int \left[\frac{1+\cos(2x)}{2}\right]dx = \frac{1}{2}x + \frac{\sin(2x)}{4} + C$$

(0,1)

$$\int \cos^1(x)\,dx = \int \cos(x)\,dx = -\sin(x) + C$$

(0,0)

$$\int \cos^0(x)\,dx = \int dx = x + C$$

So, a reduction formula works by taking one instance of a problem

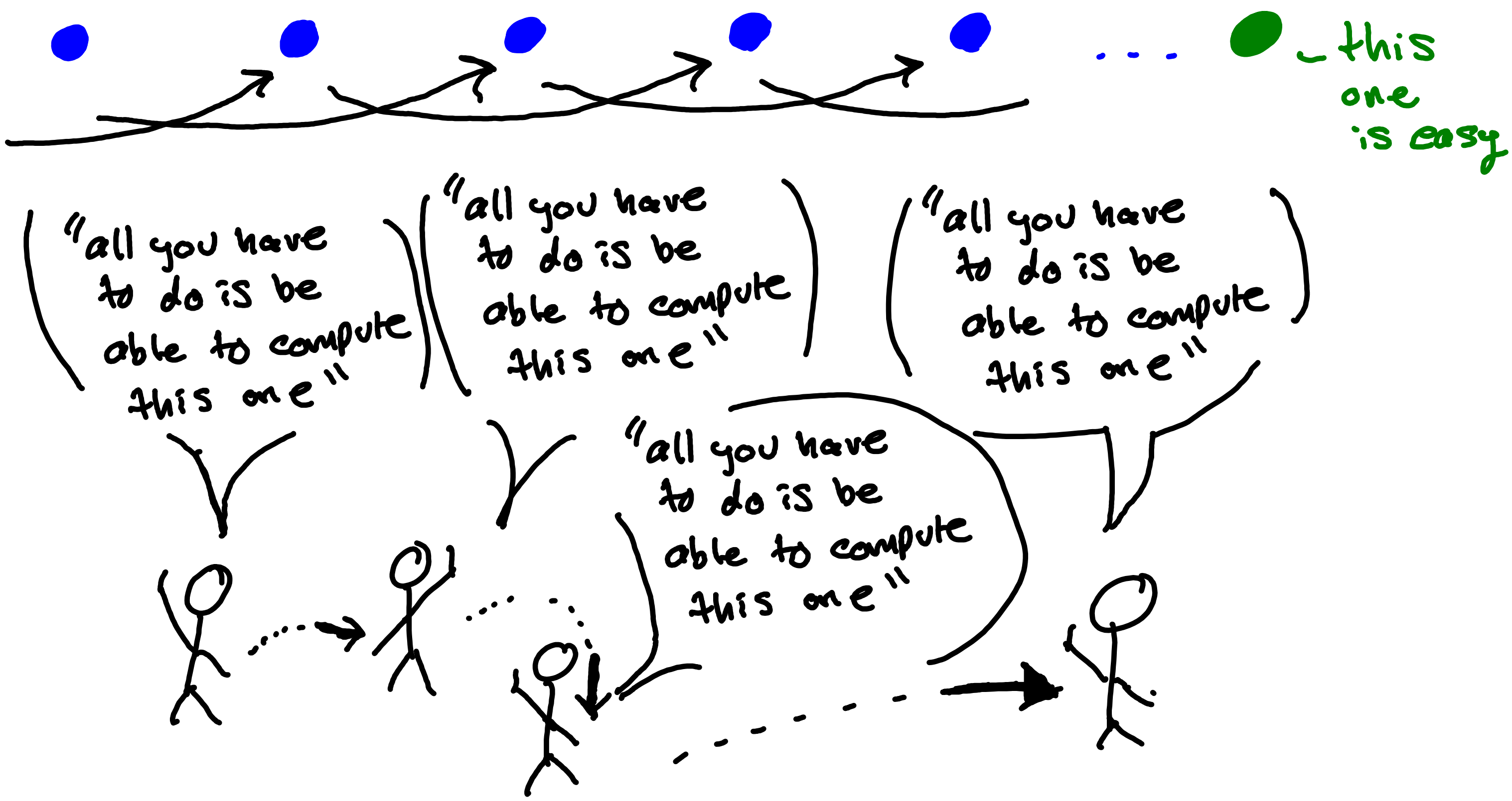
Compute $\int \cos^m(x)\,dx$

assuming $m=5$

& "solves" it by $\boxed{\text{reducing}}$ the problem to an easier version of itself (presumably a version of it you already know how to solve)

$$\underbrace{\int \cos^5(x)\,dx}_{\text{Oh, this one is easy...}} = \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\underbrace{\int \cos^3(x)\,dx}_{\text{all you have to be able to do is compute this one}}$$

$$\underbrace{\int \cos^3(x)\,dx}_{\text{Oh This one? its easy...}} = \frac{1}{3}\cos^2(x)\sin(x) + \frac{2}{3}\underbrace{\int \cos^1(x)\,dx}_{\text{all you have to be able to do is compute this one}}$$



Derivation of Cosine Reduction Formula

$$\int \cos^n(x)\,dx = \int \underbrace{\cos^{n-1}(x)}_{u}\,\underbrace{\cos(x)\,dx}_{dv}$$

$du = (1-n)\cos^{n-2}(x)\sin(x)\,dx$

$v = \sin(x)$

$\int u\,dv = uv - \int v\,du$

integration by parts formula

$$= \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,\underbrace{\sin^2(x)}_{\text{PYT}}\,dx$$

$$= \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\left[1-\cos^2(x)\right]dx$$

$\therefore$ ← this means "therefore"

n-1 copies of

$$\underbrace{\int \cos^n(x)\,dx}_{\text{one copy}} = \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,dx - (n-1)\int \cos^n(x)\,dx$$

add to both sides of equation

$$n \cdot \int \cos^n(x)\,dx = \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,dx$$

divide both sides by n & then you're done.

More Reduction Formulas You Might Try To Prove Yourself

$m \geq 3$

$$\int \sin^{m}(x)\,dx = -\frac{1}{m}\sin^{m-1}(x)\cos(x) + \frac{m-1}{m}\int \sin^{m-2}(x)\,dx$$

$$\int \sec^{n}(x)\,dx = \frac{1}{n-1}\sec^{n-1}(x)\sin(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$$

$$\int \tan^{m}(x)\,dx = \frac{\tan^{m-1}(x)}{m-1} - \int \tan^{m-2}(x)\,dx$$

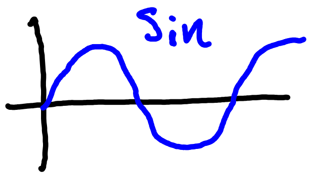
$$\int [\ln(x)]^{N}\,dx = x(\ln(x))^{N} - N\int [\ln(x)]^{N-1}\,dx$$

The pattern to keep in mind is to use integration by parts & look out for convienient times to apply trig identities (at least for the first 3 above)

Summary

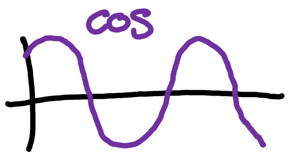
When Faced with an Integral Like $\int \sin^m(\theta)\cos^n(\theta)\,d\theta$

(i.e.



to some power

times



to a potentially different power)

The problem is begging for one of the two following substitutions

$u = \cos(\theta)$ OR $u = \sin(\theta)$

But the fact that these choices result in

$du = -\sin(\theta)\,d\theta$ & $du = \cos(\theta)\,d\theta$ (respectively)

means such a substitution will only work if there is

an odd # of sines (m is an odd number) OR an odd # of cosines (n is an odd number) (resp.)

When this does happen to be the case, we proceed as follows

convert the leftover $\underbrace{m-1}_{\text{even \#}}$ sines into cosines with PYT

$$\sin^{\text{even}}(\theta) = (1-\cos^2(\theta))^{\frac{\text{even}}{2}}$$

convert the leftover $\underbrace{n-1}_{\text{even \#}}$ cosines into sines with PYT

$$\cos^{\text{even}}(\theta) = (1-\sin^2(\theta))^{\frac{\text{even}}{2}}$$

This leaves you with an integral of the form

$$\int (1-\cos^2(\theta))^{\frac{m-1}{2}} \cos^n(\theta) \sin^1(\theta)\, d\theta \qquad \Bigg| \qquad \int \sin^m(\theta)(1-\sin^2(\theta))^{\frac{n-1}{2}} \cos^1(\theta)\, d\theta$$

Remember: Here $u=\cos(\theta)$

so $du=-\sin(\theta)d\theta$

$$\Leftrightarrow \quad -\int (1-u^2)^{\frac{m-1}{2}} \cdot u^1 du$$

Here $u=\sin(\theta)$

so $du=\cos(\theta)d\theta$

$$\Leftrightarrow \quad \int u^m (1-u^2)^{\frac{n-1}{2}} du$$

In either case we could expand out the integrand & realize that all that's left to do now is integrate a Polynomial in the variable u.

Linearity of $\int$ & Reverse Power Rule $\Rightarrow$ can integrate all polynomials.

all polynomials have a "degree" $= n$ = largest power of x that appears

$$\int (\text{polynomial in } u)\, du = \int [a_n u^n + a_{n-1}u^{n-1} + \ldots + a_2 u^2 + a_1 u + a_0]\, du$$

Fixed but unknown real #'s constants

a_i = "coefficient of u^i"

linearity of $\int$

$$= a_n \int \underbrace{u^n}_{\substack{\text{Monomial}\\ \text{of degree } n}} du + a_{n-1}\int \underbrace{u^{n-1}}_{\substack{\text{monomial}\\ \text{of degree}\\ n-1}} du + \ldots + a_2 \int \underbrace{u^2}_{\substack{\text{Monomial}\\ \text{of degree 2}}} du + a_1 \int \underbrace{u}_{\substack{\text{monomial}\\ \text{of degree}\\ 1}} du + a_0 \int du$$

"the Squaring function"

$u^0 = 1$ is a Monomial of degree zero

Reverse Power Rule

tells us how to integrate a monomial of degree n $(\neq -1)$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

Hence, $\int(\underbrace{\text{polynomial}}_{\substack{\text{Sum of constants times}\\ \text{monomials of various degrees}}}\ \overbrace{\text{in } u}^{\text{meaning the variable is } u}\ \text{of } \underbrace{\text{degree } n}_{\substack{\text{highest power of } u \text{ that appears}\\ \text{in the polynomial}\\ \text{with non-zero coefficient}}})\,du$

$$= \underbrace{\frac{a_n x^{n+1}}{n+1}}_{\text{first integral}} + \underbrace{\frac{a_{n-1}x^n}{n}}_{\text{second integral}} + \cdots + \underbrace{\frac{a_2 x^3}{3}}_{(n-1)^{st}\text{ integral}} + \underbrace{\frac{a_1 x^2}{2}}_{n^{th}\text{ integral}} + \underbrace{a_0 x}_{(n+1)^{st}\text{ integral}} + C$$

Sum of all $n+1$ constants that came from integration

$\therefore$ $\int(\text{polynomial}) = \text{polynomial of 1 degree higher}$

Linearity of the integral

$$\int[af+bg]\,dx = a\int f\,dx + b\int g\,dx$$

together w/ Reverse Power Rule

$\Longrightarrow$ we can integrate <u>ALL POLYNOMIALS</u>

The Following Strategy works when <u>n is odd</u> OR <u>m is odd</u>

$$\int \sin^m(\theta)\cos^n(\theta)\,d\theta \xrightarrow[u=\cos(\theta)]{u=\sin(\theta)} \int \begin{pmatrix}\text{polynomial}\\ \text{in } u\end{pmatrix} = \begin{matrix}\text{polynomial}\\ \text{in } u\end{matrix}$$

OR

+ pythagorean theorem to exchange even powers of sin/cos for "even powers of the other"

When Neither m nor n is odd

The <u>double angle formulas</u> decrease the values of m OR n

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

$$\underbrace{\sin^2(\theta)}_{\text{Squared!}} = \underbrace{\frac{1-\cos(2\theta)}{2}}_{\text{No squared trig functions}}$$

$$2 \xrightarrow[\text{decrease}]{} 1$$

So, you can use this idea repeatedly to get a (bunch of) integrals only involving $\underbrace{1 \sin \,\&/\text{OR}\, 1 \cos}_{\text{odd \# of sines/cosines we know how to deal with}}$

$m=0 \Rightarrow$ the above line is a recipe for the cosine <u>reduction formula</u>

$n=0 \Rightarrow$ the above line is a recipe for the Sine <u>reduction formula</u>

"reduction formulas" always come from repeated integration by parts