Basic Integration Techniques Review

Power Rule

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

Reverse Power Rule

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

For $x \neq -1$

Slogan: Shoes & Socks

(Note: $\boxed{x^n} = \int \frac{d}{dx} x^n = \int n \cdot x^{n-1}\,dx = \boxed{n \int x^{n-1}\,dx}$ so $\int x^{n-1}\,dx = \frac{x^n}{n}$)

FToC

Power Rule

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x))\,g'(x)$$

OR

$$\frac{d}{dx} = \frac{d}{du}\frac{du}{dx}$$

U-Substitution (Reverse Chain Rule)

$$\boxed{\int f'(g(x))\,g'(x)\,dx = f(g(x))}$$

$u = g(x)$ is the substitution you use

because then $du = g'(x)\,dx$

So $\int f'(g(x))\,g'(x)\,dx = \int f'(u)\,du = f(u)$
$= f(g(x))$

Where does the U-substitution formula above come from?

Basically, we just integrate both sides of the $=$ sign in the chain rule

$$\int \underbrace{\frac{d}{dx} f(g(x))\,dx}_{} = \int f'(g(x))\,g'(x)\,dx$$

FToC says these cancel

Slogans: (i) Look for a piece whose derivative appears multiplied by dx

(ii) Everything in terms of x must be expressed in terms of u (including the function, bounds & dx)

Today:

Product Rule

$$\boxed{\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)}$$

So again the formula for Integration by parts can be found by integrating this formula for the product rule

Integration by Parts

Reverse Product Rule

$$\int \underbrace{\frac{d}{dx}}_{\text{FToC Says these cancel}} f(x)g(x)\,dx = \int \left(f'(x)g(x) + f(x)g'(x)\right)dx$$

$$f(x)g(x) = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

Conclusion: We can compute integrals like

$$\int f(x)g'(x)\,dx$$

notice that u-Substitution is not guaranteed to work in this situation

by the formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

typically, we see this formula expressed as

$$\boxed{\int u\,dv = u\cdot v - \int v\cdot du}$$

$u = f(x)$

$du = f'(x)dx$

$v = g(x) + c$

$dv = g'(x)dx$

Remarks:

- Let's not forget that the formula just arrived @ can be checked for accuracy

$$\boxed{\textbf{Claim:}\quad \int u\cdot dv = u\cdot v - \int v\cdot du}$$

This means, taking the derivative of the Right Hand Side (RHS) results in the function under the integral symbol,

also known as the integrand

✔-ing the claim

Suppose we are given

$\int f(x)\cdot g'(x)\,dx$ & we solve this integral using the integration by parts formula above

hmm... we get to pick any value for this!

let $u = f(x)$ & $dv = g'(x)\,dx$

Explanation of Solution version 1

then $\frac{du}{dx} u = f(x)$ & $v = \int dv = \int g'(x)\,dx = g(x) + c$

typically we pick $c=0$ but it can help to use other values too. Be Creative but check your Answers

therefore $du = f'(x)\,dx$ & $v = g(x) + c$

So integration by parts formula gives

Explanation of Solution version 2

then $du = \frac{d}{dx} f(x) = f'(x)\,dx$ & $v = g(x) + c$

like implicit differentiation
x could be a function of something else

$$\int f(x)g'(x)\,dx = \int u\cdot dv = u\cdot v - \int v\cdot du$$

$$= f(x)\left[g(x)+c\right] - \int \left[g(x)+c\right] f'(x)\,dx$$

$$= f(x)g(x) + \underline{c\,f(x)} - \int g(x)f'(x)\,dx \underline{- c\int f'(x)\,dx}$$

$$= f(x)g(x) - \int g(x)f'(x)\,dx + c\,f(x) - c\cdot\big[f(x) + K\big]$$

another arbitrary constant of integration

$$= \underbrace{f(x)}_{u}\underbrace{g(x)}_{v-c} - \int \underbrace{g(x)}_{v-c}\underbrace{f'(x)dx}_{du} \underbrace{- c\cdot K}_{\text{package into one arbitrary constant } \mathcal{C}}$$

$\mathcal{C} \leftarrow$ "fancy c"

Therefore we arrive at the conclusion

$$\int \underbrace{f(x)}_{u}\underbrace{g'(x)dx}_{dv} = \underbrace{f(x)}_{u}\underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\underbrace{f'(x)dx}_{du} - \mathcal{C}$$

arbitrary constant of integration

$$\frac{d}{dx}(\text{RHS}) = \frac{d}{dx}\left(f(x)g(x) - \int g(x)f'(x)\,dx - \mathcal{C}\right)$$

linearity of the derivative

$$= \frac{d}{dx}\big[f(x)g(x)\big] - \underbrace{\frac{d}{dx}\int}_{\text{expect cancellation by FToC}} g(x)f'(x)\,dx \underbrace{- \frac{d}{dx}\mathcal{C}}_{\text{derivative of a constant } = 0}$$

derivative of a product of functions $\Rightarrow$ use product rule

$$= \underbrace{f'(x)g(x)}_{\text{positive}} + f(x)g'(x) \underbrace{- g(x)f'(x)}_{\text{negative}} - 0$$

positive & negative cancel out

$$= \boxed{f(x)g'(x)}$$

perfect. this is the function under the $\int$ symbol on LHS

In General:

$\int$ function d(variable) $\xrightarrow{\text{if you use integration by parts, to get}}$ some anti-derivative of function

then checking your answer by taking a derivative will always involve an application of the product rule

What Else Did we Learn?

$$\int u\cdot dv \overset{(*)}{=} u\cdot v - \int v\cdot du$$
$$= u\cdot(v-c) - \int (v-c)\,du$$

No matter what constant you replace c with

Not convinced? Simplify! or better yet, take the derivative as we did above

$$u\cdot(v-c) - \int (v-c)\,du = uv - cu - \int v\,du + \int c\,du$$
$$= u\cdot v - \int v\cdot du \underbrace{- cu + cu}_{\text{cancel out}} + K$$

$$\boxed{= u\cdot v - \int v\cdot du + K}$$

looks like we have an extra constant K but really this constant appears in the "usual formula" above (*) but it is packaged in with the indefinite integrals that always produce arbitrary constants of integration

Note: Typically, taking the c you get from integrating v to be zero

i.e. $dv = f'(x)\,dx$

$$\Big\downarrow \int$$

$$v = f(x) + \underbrace{c}_{\text{set}=0}$$

$$v = f(x)$$

I don't know of any problems in this course that require a choice of c value here to be anything other than zer

However! There are examples below where picking a specific & purposeful value of c @ the right time will simplify your work.

I recommend getting in the habit of keeping this c around for a while. If it doesn't end up being useful set it equal to zero when you are done.

The Upshot: we can replace the integral

$$\int u\,dv \quad \text{with} \quad \int v\,du$$

equivalently

$$\int f(x)g'(x)dx \quad \text{with} \quad \int f'(x)g(x)dx$$

& if we choose u & dv carefully, it is often the case that the new integral will be easier to compute

How can we ensure that our situation is improved when applying the Integration by parts formula?

old integral $\int u\,dv$ $\xrightarrow[\text{\& integrate } dv]{\text{differentiate } u}$ new integral $\int v\,du$

So it makes sense to look for

u so that the derivative, du, is SIMPLER

& dv so that the integral, v, is NOT ANY MORE COMPLICATED

(these rules will get you started, but are not the only way to integrate by parts... be creative!)

Example:

$$\underbrace{\int x\sin(x)\,dx}$$

this is the picture on the box of our puzzle

puzzle pieces

u ← Something we have to pick

dv ← once u is picked we are forced to let everything else under the integral sign to be dv

Some possible choices

(I) $u = x$
$dv = \sin(x)\,dx$

(II) $u = \sin(x)$
$dv = x\,dx$

(III) $u = x\sin(x)$
$dv = dx$

(IV) $u = 1$
$dv = x\sin(x)\,dx$

(V) $u = x\sin(x)\,dx$
$dv = 1$

not valid since we have to integrate dv but there is no differential (like dx) on the right-hand side

Let's now consider each option (I)-(IV) keeping in mind the fact that we want the new integral to be easier than the one we started with.

$$\int x\sin(x)\,dx = \int u\,dv$$

Need to compute

$$\boxed{\begin{array}{l} du, \\ v, \\ \& \int v\,du \end{array}}$$

(I) $\int x \sin(x)\,dx$

$u = x$

$\frac{du}{dx} = 1 \iff du = dx$ the derivative of x is 1 which is SIMPLER

$v = \int dv = \int \sin(x)\,dx$

$= -\cos(x) + c$

the integral of $\sin(x)$ is $-\cos(x)$ which is NOT ANY MORE COMPLICATED

So, we might try using the integration by parts formula with this choice of u & dv

$$\int x\sin(x)\,dx = x\cdot(-\cos(x)+c) - \int(-\cos(x)+c)\,dx$$

$$= -x\cos(x) + \overbrace{cx}^{\text{positive}} + \int \cos(x)\,dx \underbrace{-\int c\,dx}_{\text{negative } = -cx + K}$$

the parts involving c cancelled out no need to pick a c value

$$\boxed{= -x\cos(x) + \sin(x) + K}$$

Double check answer: If $\frac{d}{dx}(-x\cos(x)+\sin(x)+C) \overset{?}{=} x\sin(x)$ then we did not make any mistakes

$$\frac{d}{dx}(-x\cos(x)+\sin(x)+C) = \underbrace{\frac{d}{dx}(-x\cos(x))}_{\text{use product rule}} + \underbrace{\frac{d}{dx}\sin(x)}_{\cos(x)} + \underbrace{\frac{d}{dx}C}_{\text{zero}}$$

that's reassuring !!

$$= -\cos(x) + x\sin(x) + \cos(x) + 0 = x\sin(x)$$

cancel

✓

(II) $\int \sin(x)\, x\, dx = \int u\, dv$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x) \Leftrightarrow du = \cos(x)dx$

the derivative of u FAILED to get SIMPLER

$v = \int dv = \int x\,dx = \frac{x^2}{2} + c$

the integral of dv got MORE COMPLICATED

So using the integration by parts formula with this choice of u & dv IS VALID but NOT HELPFUL

$$\int \sin(x)\, x\, dx = \frac{x^2}{2}\sin(x) - \underbrace{\int \left(\frac{x^2}{2}+c\right)\cos(x)dx}$$

Hmmm.. the original integral looked easier (even if we choose $c=0$)

(III) $\int x\sin(x)dx = \int u\,dv$

$du = \frac{d}{dx} x\sin(x) = [\sin(x) + x\cos(x)]dx$

the derivative of u got MORE COMPLICATED

$v = \int dv = \int dx = x + c$

the integral of dv got MORE COMPLICATED (even with $c=0$)

So we will be further from having a solution if the integration by parts formula is used with this choice of u & dv

(IV) $\int 1 \cdot x\sin(x)\,dx = \int u\,dv$

$du = \frac{d}{dx} 1 = 0 \cdot dx$ The derivative of u got SIMPLER

$v = \int dv = \underbrace{\int x\sin(x)\,dx}$

oops, we can't find an expression for v without solving the original integral first

Conclusion: it NEVER helps to take $u = 1$ for integration by parts.

Similarly, it NEVER helps to take $u = x$ for a u-substitution (because you will end with exactly the same integral you started with)

Look out for "invisible dv's"

Compute $\int \ln(x)\,dx$ using integration by parts

Hint: $\frac{d}{dx}\ln(x) = \frac{1}{x}$

What are our options?

well, we know $u=1$, $dv=\ln(x)dx$ won't work

So we're forced to take $u=\ln(x)$, $dv=dx$

Recall: $$\int u\,dv = uv - \int v\,du$$

$$\int \ln(x)dx = \ln(x)\underbrace{\int dx} - \int \underbrace{\left(\int dx\right)}\underbrace{\frac{d}{dx}\ln(x)}$$

need to compute this

& this

from the hint

$$\left(du = \frac{d}{dx}\ln(x) = \frac{1}{x}dx \right) \Longleftrightarrow \left(\frac{du}{dx} = \frac{d}{dx}\ln(x) = \frac{1}{x} \quad \text{Therefore} \quad du = \frac{dx}{x} \right)$$

hint

equivalent way of showing work

$$v = \int dx = x + C$$

notice that numerator = denominator if $c=0$

$$\Rightarrow \int \ln(x)dx = (x+c)\ln(x) - \int \underbrace{(x+c)\cdot\frac{1}{x}}dx$$

$$= \underbrace{x\cdot\ln(x)} + \underbrace{c\cdot\ln(x)}_{=0 \text{ if } c=0} - \underbrace{\int \underbrace{\frac{x}{x}}_{=1}dx}_{=x+K} - \int \frac{c\,dx}{x}$$

get rid of this second integral by setting $c=0$

$$\boxed{= x(\ln(x)-1)+K}$$

Checking our answer: $\frac{d}{dx}x(\ln(x)-1)+c \stackrel{?}{=} \ln(x)$

$$\frac{d}{dx}x(\ln(x)-1)+c = \underbrace{\frac{d}{dx}x(\ln(x)-1)}_{\text{use product rule}} + \underbrace{\frac{d}{dx}C}_{\text{zero}}$$

$$= \ln(x)-1 + x\left(\frac{1}{x}-0\right)+0 = \ln(x)-1+1 = \ln(x) \checkmark$$

Similar Examples

a) $\int \ln(x+1/2)\,dx$

b) $\int 8\cdot\ln(2x)\,dx$

c) $\int 7\cdot\ln(3x+2)\,dx$

Fixed but unknown constants

d) $\int [\alpha\cdot\ln(\beta x+\gamma)+\delta]\,dx$

Solutions: We will start to see the advantage of the $+c$ now

numerator = denominator when $c=1/2$

a) $\int \ln(x+1/2)\,dx = \ln(x+1/2)(x+c) - \int \frac{x+c}{x+1/2}\,dx$

$u=\ln(x+1/2)$ $\quad v=x+c$

$du=\frac{1}{x+1/2}\,dx$ $\quad dv=dx$

Setting $c=1/2$ (as we are free to) we get

$$= \ln(x+1/2)(x+1/2) - \int \underbrace{\frac{x+1/2}{x+1/2}}\,dx$$

the intende po int of choosing $c=1/2$ is to be able to cancel this fraction right now

$$= \ln(x+1/2)(x+1/2) - \int 1\cdot dx$$

$$\boxed{= \ln(x+1/2)(x+1/2) - x + K}$$

Q: what happens if you simply set $c=0$? is it obvious that the answer has not changed? try it

b) $\int 8\cdot\ln(\underbrace{2x}_{u})\underbrace{dx}$ (Decoy! No integration by parts needed if you know that $\int \ln(t)\,dt = x[\ln(x)-1]+K$

$u=2x$

$du=2dx$

$dx=\frac{du}{2}$

Then $\int 8\cdot\ln(2x)\,dx = 8\cdot\int \ln(u)\,\frac{du}{2}$

$$\boxed{=4\cdot[x(\ln(x)-1)+K]}$$

But integration by parts w/ $u=8\ln(2x)$, $dv=dx$ works too c)

$$\int 7\cdot \ln(3x+2)\,dx$$

$$= 7\cdot\int \underbrace{\ln(3x+2)}_{u}\underbrace{dx}_{dv}$$

$$= 7\cdot\left[\ln(3x+2)\cdot v - \int v\,du\right]$$

$$v = \int dv = \int dx = x + c$$

$$du = \frac{d}{dx}\ln(3x+2) = \frac{1}{3x+2}\cdot\frac{d}{dx}(3x+2)$$

$$= \frac{3}{3x+2}dx$$

$$= 7\cdot\left[\ln(3x+2)(x+c) - \int \frac{(x+c)\cdot 3}{\underbrace{3x+2}}dx\right]$$

what c value makes this cancel?

i.e. when does

$3(x+c) = 3x+2$?

$\quad\;\;\; \| $

$3x + 3c$

$\Leftrightarrow \; 3c = 2 \;\Leftrightarrow\; c = 2/3$

$$= 7\cdot\left[\ln(3x+2)(x+2/3) - \int 1\,dx\right]$$

$$= 7\cdot\left[\ln(3x+2)(x+2/3) - x + K\right]$$

$$\boxed{= 7x\ln(3x+2) + \frac{14}{3}\ln(3x+2) - 7x + \underbrace{\tilde{K}}_{=7K}}$$

d)

$$\int\left[\alpha\cdot\ln(\beta x+\gamma)+\delta\right]dx$$

$\alpha, \beta, \gamma, \delta =$ fixed but unknown constants

$$= \underbrace{\alpha}\underbrace{\int \ln(\beta x+\gamma)\,dx}_{(**)} + \delta\underbrace{\int 1\cdot dx}_{=x+C}$$

remember the parts in red later

focus on blue integral for now

$$(**) = \int \ln(\beta\cdot x+\gamma)\,dx$$

$u = \ln(\beta x+\gamma) \qquad v = x + c$

$du = \frac{\beta}{\beta x+\gamma}dx \qquad du = dx$

chain rule

$$= \ln(\beta x+\gamma)(x+c) - \int\frac{\beta(x+c)dx}{\underbrace{\beta x+\gamma}}$$

integration by parts

for what c value does this cancel out

i.e. solve for c if

$\beta(x+c) = \beta x+\gamma$

$\cancel{\beta x} + \beta c = \cancel{\beta x} + \gamma$

$\boxed{c = \gamma/\beta}$

$$(**) = \ln(\beta x+\gamma)(x+\gamma/\beta) - x + K$$

Final Answer

$$\boxed{\alpha\ln(\beta x+\gamma)(x+\gamma/\beta) - x + K}$$

don't take my word for it

take a derivative instead

Sometimes, one has to use Integration By Parts multiple times to get an answer

<u>Example</u>:

$$\int \underbrace{x^2}_{u} \underbrace{\sin(x)dx}_{dv} \quad \left(= uv - \int v du\right)$$

$du = 2x dx$ ← Got SIMPLER

$v = \int \sin(x)dx = -\cos(x)$ ← NOT MORE COMPLICATED

(Set $C=0$ for clarity of presentation of this example)

$$\Rightarrow \int x^2 \sin(x)dx = -x^2\cos(x) - \int (-\cos(x)\cdot 2x dx)$$

$$= 2\underbrace{\int x\cos(x)dx}_{} - x^2\cos(x)$$

ok, so the integral got simpler but it isn't immediately obvious what the new integral is equal to

So, use integration by parts again on this simpler integral

$$\int x\cos(x)dx = \int u dv$$

$du = dx$

$v = \int \cos(x)dx = \sin(x)$

(again take $C=0$ for this example so we don't have to look @ a mess)

Hence $\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx$

$$= x\sin(x) + \cos(x) + C$$

meaning, we now can answer the problem we started with

$$\int x^2\sin(x)\,dx = 2\int x\cos(x)\,dx - x^2\cos(x)$$

$$\boxed{= 2x\sin(x) + 2\cos(x) - x^2\cos(x) + C}$$

Q: Can you find a function whose integral would require 3, 4, 5, ... or n uses of Integration by Parts?

Practice Problems

a) $\int \arctan(x)\,dx$

Hint: $\frac{d}{dx}\arctan(x) = \frac{dx}{1+x^2}$

b) $\int (x+4)e^x\,dx$

c) $\int x^2 e^{2x}\,dx$

d) $\int \sqrt{x}\ln(x)\,dx$

Strategies

- Puzzle where du is simpler than u & v is not more complicated than dv
- delay picking a value for C
- Invisible dv
- Use integration by parts multiple times

Another Clever Way to Use Integration By Parts

("integration by parts swallowing itself")

$$\int e^x \cos(x)\,dx$$

We give it a shot by picking u & dv somehow & applying the formula

c=0 here so the example is not cluttered w/ extra stuff

$$u = e^x \qquad v = \sin(x)$$

Compute

$$du = e^x\,dx \qquad dv = \cos(x)\,dx$$

compute

Nothing got simpler but definitely nothing got more complicated either. Let's see what we get

$$\int u\,dv = uv - \int v\,du$$

$$\int e^x \cos(x)\,dx = e^x \sin(x) - \underbrace{\int \sin(x) e^x\,dx}_{\text{Hmm... although different this looks about the same as the function we started with}}$$

As we saw, sometimes it can be good to integrate by parts more than once. So let's not be discouraged & simply continue using integration by parts

for the same reason c=0 again

$$\int \sin(x) e^x\,dx \qquad u = e^x \qquad v = -\cos(x)$$

$$du = e^x\,dx \qquad dv = \sin(x)\,dx$$

$$\int \sin(x)e^x\,dx = e^x(-\cos(x)) - \underbrace{\int (-\cos(x))e^x\,dx}_{\text{but wait! this looks familiar...}}$$

Original Goal

$$\int e^x\cos(x)\,dx \underset{\substack{\uparrow \\ \text{Integration} \\ \text{by Parts}}}{=} e^x\sin(x) - \int \sin(x)e^x\,dx$$

another use of integration by parts

$$= e^x\sin(x) - \left[-\cos(x)e^x - \int -\cos(x)e^x\,dx\right]$$

$$= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx$$

Which means

$$\int e^x\cos(x)\,dx = e^x(\sin(x)+\cos(x)) - \int e^x\cos(x)\,dx$$

Add $\int e^x\cos(x)\,dx$ to both sides of the equation

$$2\int e^x\cos(x)\,dx = e^x(\sin(x)+\cos(x))$$

$$\Longrightarrow \boxed{\int e^x\cos(x)\,dx = \frac{e^x(\sin(x)+\cos(x))}{2} + C}$$

Q: What happens if we do the same initial substitution

$$\int \underbrace{e^x}_{u} \underbrace{\cos(x)\,dx}_{dv} = e^x \sin(x) - \int \underbrace{\sin(x) e^x\,dx}_{}$$

then change up our choice of substitution here?

i.e. Now $u = \sin(x)$ $\quad dv = e^x dx$

$du = \cos(x)dx$ $\quad v = e^x$ (take $c=0$ for simplicity)

then we end up with

$$\int e^x \cos(x)dx = e^x \sin(x) - \left[e^x \sin(x) - \int e^x \cos(x) dx \right]$$

Simplifying ..

$$\int e^x \cos(x) dx = (\underbrace{e^x \sin(x) - e^x \sin(x)}_{\text{Cancel out}} + \int e^x \cos(x) dx$$

$$\Longleftrightarrow \quad \int e^x \cos(x) dx = \int e^x \cos(x) dx$$

We get a different way to express the desired integral in terms of itself, but this one is not as helpful as the other. This one says something obvious that we already knew.

A: Need to stick w/ the same choice of u to make progress on this problem.

Q: What if we started w/ the other choice of assignment of u & dv?

Short answer - the "same thing" happens as above

$\boxed{\text{TRY BOTH \& SEE}}$

By Parts #1

$u = \cos(x)$
$du = -\sin(x)dx$
$v = e^x$
$dv = e^x dx$

"Same"

By Parts #2

$u = \sin(x)$ $\quad v = e^x$
$du = \cos(x)dx$ $\quad dv = e^x dx$

✓ Helps Solve Problem

"opposite"

$u = e^x$ $\quad v = -\cos(x)$
$du = e^x dx$ $\quad dv = \sin(x)dx$

✗ Will Not Help

One Last Thing to be aware of

When computing a DEFINITE INTEGRAL the integration by parts formula looks like

$$\int_a^b u\,dv = \underbrace{uv\Big|_a^b}\; - \int_a^b v\,du$$

this means plug in b to the expression uv & subtract from it the value you get when plugging a into uv

For Example.

$$\int_1^5 \frac{\ln(x)}{x^2}\,dx$$

$u = \ln(x)$ $\qquad v = \int x^{-2}\,dx = -x^{-1} = \frac{-1}{x}$

$du = \frac{1}{x}dx$ $\qquad dv = \frac{1}{x^2}\,dx = x^{-2}\,dx$

$$\int_1^5 \frac{\ln(x)}{x^2}\,dx = -\frac{\ln(x)}{x}\Big|_1^5 - \int_1^5 -\frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$= \frac{-\ln(5)}{5} + \frac{\ln(1)}{1} + \int_1^5 x^{-2}\,dx$$

$$= \frac{-\ln(5)}{5} + 0 - x^{-1}\Big|_1^5 = \boxed{\frac{-\ln(5)}{5} - \frac{1}{5} + 1}$$